# HANDBOOK TO SMITH'S ARITHMETICS

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# HANDBOOK

TO

# SMITH'S ARITHMETICS

BY

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# PREFACE

This handbook has been prepared in the hope that it will be helpful not only to teachers who are using the author's series of arithmetics, but to those who are training others to enter the profession, and to all who are interested in the modern teaching of this most ancient of all the sciences. Such a book would signally fail to fulfill its best mission if it were looked upon as a mere guide for a single series of text-books, and hence in its preparation there has been kept in mind the broader view of general helpfulness to the profession at large.

In the Introduction a few suggestions are given on the general question of method, the limited space at the author's disposal not allowing of a more detailed treatment. The manifest purpose in these suggestions is to show the danger of extremes in theory, and the necessity of knowing the best that the world has produced in relation to the teaching of arithmetic. While a few ideas are also advanced as to the conduct of the class and the use of the text-book, it is felt that no book can or should assume to furnish individuality to a teacher. Therefore it is recognized that these ideas, as well as those relating to the presentation of the different topics, to the explanation of various operations, to the division of the work of the successive years, and to the correlation of arithmetic with the other subjects of the curriculum, will give way to the ideas of the teacher as occasion demands.

The handbook also includes a good modern working course of study, not running to any extreme, not based on the circumscribed view of any one person, but prepared after a long and careful study of the best courses that are at present in use in the educational centers of the United States. To have such a curriculum at hand, to feel that it is usable, and to see that it covers the demands of such great examining bodies as the Regents of New York State, cannot fail to be helpful to all who are in touch with the teaching of the subject.

It is hoped that the suggestions as to the treatment of the various topics will be of service to all who give instruction in arithmetic, whether they are using the author's works or not. He would be too narrow to write worthily who should feel that his own text-books were the only ones to have merit, or should seek to withhold from users of other works such assistance as he might be able to give. Accordingly the teacher will find in this handbook numerous discussions of the educational values of various topics, hints as to methods of presenting the different subjects, much supplementary matter of service to teachers, although too advanced for the pupils, and considerable interesting historical information, as well as a running comment upon the successive chapters of the author's text-books. Certain topics that are still practical in some sections of the country, but are no longer common in business, such as annual interest in connection with partial payments, and others that are often demanded on the ground of mental discipline, such as the long-division form of the greatest common divisor, find their place in such a handbook rather than in an arithmetic for general use.

DAVID EUGENE SMITH

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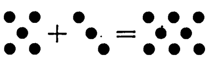
# HANDBOOK TO SMITH'S ARITHMETICS

## INTRODUCTION

### GENERAL SUGGESTIONS

**Primary methods.** To a diminishing number of teachers the text-book is a fetich, and to have the pupils "go through" the printed problems and "get the answers" is the aim of the year's work. Of notably higher rank is the teacher who recognizes the book as furnishing most of the material, and uses good common sense in leading the children with fair intelligence through the difficulties of computation. Then comes a third grade of teacher, not always as safe as the one just described, whose ambition it is to find some single method that shall be a panacea for all the mathematical ailments of childhood. The method selected may be some extreme of an idea suggested by Pestalozzi (about 1800), such as emphasizing the unit, or learning all the operations to ten before learning the numerals, or making practically all of the problems abstract. It may be Von Busse's idea (1786) of the *Zahlenbilder* (number pictures), in which a whole arithmetic is made of work like



It may be Krancke's concentric-circle plan (1819), or Grube's adaptation of it (1842), with various features that rendered the method hopeless. Again, it may be the Kaselitz method of always

emphasizing number as an operator; or the Tanck and Knilling method of seeing in the subject of counting the only hope for primary arithmetic; or the ratio plan, which properly recognizes number as a ratio, but improperly attempts to have little children appreciate the fact. It may be the measuring method, based on the idea that all number is the result of measuring, and that therefore children should measure everything within reach. Instead of emphasizing the operations with abstract numbers, as Pestalozzi did, we may let the pendulum swing to the other extreme, and give to the children "no problems without content," in other words, no problems with abstract numbers, the result being that the children's minds are always occupied with the solution of the problems, so that they never concentrate on the operations alone,—a result which tends to make them poor computers. These plans are mentioned merely for the purpose of showing how easy it is to find a method, to go to an extreme, to try to do with children what no one would think of doing with himself. One of the easiest things in teaching arithmetic is to create a method.

Teachers of this third type are rarely without a following. Their enthusiasm and vigor carry success in spite of their extreme views. They say, "Must not my method be right, with such results?" But invariably the results are due to the personality of this type of teacher, not to the method. They do not harm their own classes, for they have strength to overcome the obstacles which they themselves create, but they do harm the pupils of weaker teachers, who make the effort to follow their example.

Then comes a fourth type of those who lead our children through the mazes of their early school days. Teachers of this type are wedded to no single method, and yet they

recognize that in each there is a grain of truth which they should appreciate. Hence they seek to hold to the best that is in the old teaching, while sympathetically seeking the best that is in the suggestions of the extremists. They see that the ratio idea has merit, that it is usable at certain times and for certain purposes, but they do not seek to employ it merely for the sake of doing so. They see the value of counting in developing the tables of addition and of multiplication, and they recognize the interest of rhythm as it appears in this method; but to count by 7's from 3 to 150, or backwards by 6's from 150 to 0, is to carry a good thing to an extreme that will not appeal to their common sense.

It is the aim of Smith's *Primary Arithmetic* to help teachers to rank with this fourth type; to suggest the best that is in all methods; to use the counting plan where it is helpful and not otherwise; to bring in the ratio idea, without the name, where it is of value, but to carry it to no extreme; to suggest a variety of objective work where such work is needed, but not to fetter the teacher with any one set of objects, nor to weaken the pupil by continuing the objective work after its purpose has been fulfilled.

**When should arithmetic be begun?** Before Pestalozzi's time arithmetic was usually taught only to boys who were going into business. They could already read and write, and were ready to take up the old arithmetic with its unpsychological arrangement. Pestalozzi saw that arithmetic could easily be taught to children in the first grade, and that they had a desire to learn it. He therefore had his pupils begin it as soon as they entered the school, and with such success that this feature was generally adopted in Europe and America. Of late years the pressure of new subjects upon the curriculum has been so great that some

teachers are advocating the postponement of arithmetic until the second or third grade, claiming that the child's attainments will be just as great by the time he finishes his course. Even then they would have number facts taught incidentally in the first year or two. But this plan is not a good one educationally, for several reasons: (1) A child on entering school has the same desire to know about numbers that he has to know about reading, or writing, or any other subject in the course. He wishes to count in his games, to know about simple purchases, and to read numbers occurring in his books. (2) The postponement is not justified on the ground of difficulty, since this is no more marked than with other subjects. (3) Unless the subject is definitely in the curriculum it will be neglected. Teachers have too many demands upon their time and energies to arrange for incidental instruction that shall cover the needs of children in number work. There is just as much reason in saying that reading should be taught incidentally, as to say that arithmetic should be so taught. Each is necessary, each is interesting, and for each the child has a taste. (4) Psychologically, when the child enters school he is entirely ready for the memorizing of the most elementary number facts.

Arithmetic should therefore be taught in the first school year as a topic, with certain definite work laid out, the nature of this work being in harmony with the needs of a child six years of age.

**When should a book be introduced?** The experiment of introducing a book in the first grade has been repeatedly tried, and also of going to the other extreme of having no book at all in the primary grades. The first of these plans puts a book into the hands of children who do not know how to read, and therefore makes the subject seem far

harder than it really is. The second plan wastes the time and energy of pupil and teacher to no purpose, since the book would be a great labor-saving device, furnishing material carefully selected and properly arranged for use. Between these two extremes naturally lies the common-sense mean. It stands to reason that a book should be introduced as soon as a child can use it profitably. When this is, depends upon the school. Certain sections of the country, certain wards of a city, surround children with home influences that lead them to read earlier than children from other sections. In the majority of schools a child is ready for a book in arithmetic by the middle of his second school year, the teacher helping him as may be necessary with the reading of his advance lesson. In other schools the use of a book may reasonably be postponed to the third year. The wisdom of the teacher must determine this question.

**What should be the work of the first school year?** Substantially that covered in pages 1–31 of Smith's *Primary Arithmetic.* The object of inserting this in a text-book intended for the second or third grade is that a pupil may begin the work with material that is familiar to him, so that he may acquire confidence. The object of giving so few pages to this chapter is that a more extensive treatment, the child having mastered the mathematical part, would create a distaste for the subject. This first year's work is the number space from 1 to 12 inclusive. Some teachers prefer to stop at 10, thus thinking to emphasize the decimal feature. This is a mistaken idea, however, because the decimal feature is never appreciated until a child begins to count by tens. Others prefer to stop at 20, claiming that a child needs numbers above 10 in the first year. It is perhaps as well to look with favor upon the

children's counting as far as they wish, limited by 100, since there is a pleasure in counting, and counting is always needed beyond the limits of the operations in hand. But for purposes of operation and of problem solving 12 seems a good limit. This number is used very early in measuring lengths (the inch and foot), and telling the time by the clock, and its use affords an opportunity to introduce in a simple way the reading and writing of two-figure numbers.

Within this number space the children should learn how to count objects and groups of objects, how to add and subtract, the meaning of multiplication and its relation to addition, and the parts of a single object (the first natural notion of a fraction being part of some one thing) and of a group of objects (the second natural notion of fractions). The inch, foot, and yard, the cent, dime, and dollar (as ten dimes) should be familiar by actual use of the measures and of real or toy money.

**Oral and written problems.** In the earlier grades the oral work predominates; in the later grades the written work takes the more time. All that a text-book should do is to suggest a line of oral problems for the teacher and furnish most of the written problems for the pupil. The best oral problems are those that come easily and naturally from the mouth of the teacher, that are given with vivacity, that either are abstract or, when concrete, are related to the child's actual interests and needs. The best correlation is that which appears in oral problems given by the teacher, touching the children's home life, their games, the measures they have made, and their other studies. Many times the number of oral problems set forth in any book should be given by the teacher. Indeed, a book fails of accomplishing the best results if it gives too

many oral examples in topics where a teacher can better give them. Abstract seat work should always be delayed until the pupil can do it without mistakes. Premature work of this kind induces slothful habits and leads to inaccuracy.

**Nature of the drill work.** The drill work should be both oral and written. A teacher who neglects either phase of this work will find that results are unsatisfactory. About five minutes a day devoted to rapid oral work are sufficient to keep grammar-school pupils in practice. As already stated, in the primary grades the work is chiefly oral. There should also be a definite amount of rapid written work every day. If this is carried all through the grammar school, pupils will not show, as is too often the case, an inexcusable inability to perform rapidly and accurately the fundamental operations. The written drill work should be upon abstract as well as concrete examples. The former should be solved within a definite time limit, which the teacher should place upon the class. This means that the class should see how many problems can be solved in five minutes, or ten minutes, or a half hour. Of course it will not be expected that all pupils will solve the same number, because some are naturally more rapid than others. All should, however, solve a reasonable number, and accuracy rather than rapidity should be demanded. In the concrete problems it is less satisfactory to place a time limit, for the reason that the pupil has not only to perform the calculation but is to interpret the problem as well. A statement that will strike one pupil as entirely clear may be somewhat puzzling to another. Nevertheless in all such work the element of time should be considered as far as possible.

**How to use a text-book.** The oral problems are not intended to be read to the class by the teacher. The

children should read them and at once give the answers. In general, this reading should be done aloud, but in rapid work it is often better to have the class read the problems to themselves, simply giving the answers orally. Oral problems dictated by the teacher should usually be original ones instead of those in the text-book. This is the best way to make the work seem real and interesting to the class. In the written work, both abstract and concrete problems should be given, as in the author's text-books. A teacher who neglects the abstract work is like an instructor in piano playing who neglects the finger exercises. One who neglects the concrete problems is like a pianist who neglects the playing of definite tunes. In general it is better to use a text-book without the answers, having a set of answers upon the teacher's desk for consultation when the problems are not easily checked. In some schools, however, it will be found better to use the books with the answers. Whenever the text-book gives a set of problems relating to some of the industries of the country, it is desirable to correlate that work with the geography, if this can reasonably be done.

**How to mark papers.** There is only one test for a question involving a single operation. Either the answer is right or it is wrong. If the problems require some interpretation, a teacher may properly mark both for operations and for method; that is, a pupil may perform his operations correctly, but may have misinterpreted the meaning of the problem. In that case some credit may properly be given for the correct operation. In general, however, papers in arithmetic should be marked, as they are in business, by the accuracy of the result. If the result is wrong, the paper is wrong. The converse of this statement is not true, for the result may be right, and yet

the paper may be justly criticised for its slovenly appearance and the inaccuracy of the forms used. Where a time limit has been set, and a class has been given twenty minutes to solve as many problems as possible, teachers must use their judgment as to marking pupils who are naturally slow. If their work is accurate, and they have done a reasonable number of examples, they are entitled to credit and should receive commendation.

**Follow the business methods.** There are many eccentricities to be found in the schoolroom that have no warrant from the business standpoint. A teacher who is not in close touch with business customs is often tempted to invent forms of operations or methods of solving, and to impose these on the class. But we should always remember that unless there is such a decided superiority in some new form of solution as to warrant its introduction, the solutions of the business world should be adopted. On this account children in the primary grades should not be required to put their work in steps. After they have reached the grammar grades, and their habits have become somewhat fixed, it is quite legitimate to have them use the step forms, because these give the analysis more clearly.

**Formal analysis.** There was at one time a custom in the schools to have young children learn formal analyses as given by the teachers. They rarely understood the meaning of these forms, and as a result they were of no particular value to them. In the earlier grades children should give simply the answer to a problem; later, they may be asked how they found this answer, in which case they should reply in their own simple language. The formal analysis of problems, given in step form and explained logically, should not be attempted in any serious way before the fifth grade.

**The nature of the problems.** The applied problems should relate to real conditions as far as possible, and when teachers seek to add to the interest in the work by giving examples from outside the book, they should correlate the arithmetic with the other studies of the pupils and with local affairs. Some writers object to having the children apply their arithmetic to problems relating to our own country, to our industries, to our actual business life, to the games of pupils, and to the purchases of the home; but this is only because they have not the energy to prepare problems of this kind for their own books. They offer pages of uninteresting, meaningless examples that are no better than so much abstract work, and which might profitably give place to the latter. It is the aim of Smith's arithmetics to give plenty of abstract work and plenty of genuine, interesting, American, twentieth-century, concrete problems, and work of this character is recommended to progressive teachers.

# CHAPTER I

## THE FIRST SCHOOL YEAR

### The Course of Study

**The leading mathematical feature.** The addition tables are begun in this grade, and they constitute the leading feature of the year's work.

**Number space.** For operations, 1 to 12; for counting and writing numbers the limit may be extended to 100.

**Addition.** The addition of any two numbers whose sum does not exceed 10. Some teachers prefer to take the addition table of 1's, 2's, 3's, and 4's, to $9 + 4$, during the first year. The arrangement is immaterial provided the addition table through 9's is learned before the text-book is placed in the hands of the pupil.

**Subtraction.** The inverse cases of addition. For example, the fact that $5 + 4 = 9$ should lead to the statements that $9 - 5 = 4$, and $9 - 4 = 5$.

**Multiplication.** Little attention should be given to this subject in the first grade. It may incidentally be introduced and related to addition, as in $2 + 2 + 2 = 3$ times 2.

**Division.** This is considered in this grade only under the form of fractional parts. Children at this time have no need to know how many 3's there are in 9, but they do need to know $\frac{1}{2}$ of 4.

**Fractions.** It is well to introduce $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$ in the first grade, because children hear these fractions used and need to know their significance. Three facts should be

considered, say with respect to $\frac{1}{2}$: (1) $\frac{1}{2}$ of a single object; (2) one object $\frac{1}{2}$ as large as another; (3) $\frac{1}{2}$ of a group of objects. The first is the most natural idea of $\frac{1}{2}$, — the breaking of something into 2 equal parts; the second is the ratio idea, which should not be allowed to overshadow the other two, and should not concern itself with ratios as an end; the third is the more mature idea of $\frac{1}{2}$ of a group.

**Tables of denominate numbers.** Length: inch, foot, yard; value: cent, nickel, dime, dollar (as ten dimes); capacity: pint, quart. Actual measures should be placed in the hands of the children, and used by them. Other terms, like *pound*, *week*, *mile*, and *gallon*, should be used as necessary, but should not be given in the tables.

**Objects.** These are to be used as necessary, but abandoned as soon as they have served their purpose.

**Symbols.** Children should know the meaning of expressions like $6 + 3 = 9$, $9 - 6 = 3$, but in general they should use the column arrangement with the signs + and −, and then without. They thus visualize the forms that they will actually use in all their computations.

$$\begin{array}{r} 6 \\ +3 \\ \hline 9 \end{array} \qquad \begin{array}{r} 9 \\ -6 \\ \hline 3 \end{array}$$

**Technical expressions.** While it is proper to introduce the subjects by reading $6 + 1$, "six and one," and $6 - 1$, "six less one," the words "plus" and "minus" should soon come to be used. They are the technical terms of the science and have no difficulties for the child.

**Nature of the problems.** In every grade the problems should appeal to the interests of the children and to their home life and immediate needs. An example is not necessarily a concrete or applied problem simply because it relates to dollars or cattle; it must touch child life in order to be genuinely concrete.

**Abstract computation.** It is a serious error in teaching to neglect abstract drill work. The pupil's attention must be concentrated on the process, not on the application or on the logic of a problem, if he is to fix the number facts in his memory. This drill work is largely oral in the first grade, but even here a considerable amount of written work should be required.

**Text-book.** It is not advisable to attempt to use a text-book in this grade. The references in this chapter are, however, to Smith's *Primary Arithmetic*, Chapter I, which reviews this work.

## Details of the First Year's Work in Arithmetic

**Counting** (Page 1). Children like to count, and counting is the most important single feature in all arithmetic. We count and read numbers a thousand times where we divide once. Drill on counting things, but not always the same class of things. Then drill on counting groups, thus paving the way for fractions, for the tables, and for counting by tens in the second year. All of arithmetic rests on counting. If we count by 2's, beginning with 2, we have 2, 4, 6, 8, 10, which is really the beginning of the multiplication table of 2's. If we begin with 1, we have 1, 3, 5, 7, 9, 11, which, with the series above, gives the entire addition table of 2's. These features are not taken up very extensively in the first year, but the matter is mentioned here as showing the importance of counting by various numbers.

**Lengths** (Page 4). The inch, foot, and yard are the measures of length appropriate to the first year. Measuring can be carried to an unwarranted extreme, but a little of it each day or two is valuable. Notice the value of estimating lengths, as shown on page 5. In all measuring work

the children should use the units of measure, so as to visualize the yard, the foot, and the inch.

**Addition.** The children should learn in this year the number facts given on page 7. These facts are learned by actually counting objects, inch cubes forming as good a medium as any, although other objects should be freely used. Of course this is a review, and the details of presenting the facts of this table (page 7) cannot be given in the textbook. In general, it is better to have the children visualize the addition in the vertical form shown on page 9, and to drill upon that rather than upon the horizontal form shown on page 7, because we almost always add in columns. It is necessary, however, that both forms should be given and be understood, because the second is so often seen in print. Children should use both the word "plus" and the word "and," each being a word in common use. The circles given on page 8 are helpful for a change, but we should never forget that these are not as good forms for drill as those on page 9, because we need to visualize the latter for our column additions, while we never meet the former in computation.

**Subtraction** (PAGE 11). The child should be prepared for this subject by a large amount of drill like that given on page 10. Primary arithmetic does not assume to dictate to a teacher as to how subtraction shall be presented. Ever since the world's first arithmetic was printed there has been a continued debate upon this subject, and it is considered later in this manual. At present it is merely necessary to ask whether the child shall be led by the symbols $7 - 4$ to think of 4 objects taken away from 7 objects, or of the number of objects which must be added to 4 to make 7. Our words "take 4 from 7" suggest the former plan, but the fact that this requires the learning of a subtraction table, while

the other depends only on the addition table, has made the second plan very popular with the best class of teachers.

$$\begin{array}{r}7\\-4\\\hline\end{array}$$

If a child comes to think of these symbols as demanding the number which together with 4 makes 7, the mental image of $3 + 4 = 7$ comes to mind, and he writes 3 below the line. Computers find this the quicker way, and we all use this plan in making change. If, therefore, a child has not already learned the other plan of subtracting, this is the better for him. It is inexcusable, however, to bother him by changing from one plan to another as he goes from grade to grade. Let a school adopt one plan and adhere to it. The text-book admits of either, not only to avoid confusing children who have learned a plan already adopted by a school, but because all teachers find it more satisfactory to explain this process orally without the help of books. For drill upon tables like that upon page 14, teachers usually find it helpful to make sets of cards having the numbers printed on one side, as here shown, and their difference written on the opposite side. The numbers should be printed large enough to be easily seen across the room.

$$\boxed{\begin{array}{c}8\\ \underline{3}\end{array}}$$

The same plan is useful in drill upon addition, multiplication, and division.

**Multiplication** (PAGE 15). This topic is usually treated very briefly in the first grade. Children need only to have the idea, with a little drill upon such simple problems as relate to their daily life. The subject should be approached from the standpoint of addition, and the triangular arrangement of numbers on page 15 is exceedingly helpful for this purpose and for various number drills. As to writing the statement of multiplication, the old English form was $\$3 \times 2 = \$6$. At present the tendency is decidedly in favor of writing and reading the multiplier first, as $2 \times \$3 = \$6$.

This is much less confusing to children, and it is used by the majority of our best writers and teachers.

**Fractions** (PAGE 17). A child naturally approaches fractions from the standpoint of parts of one object. He breaks a thing in two equal parts, and has the notion of half. This is the world's primitive fraction and the way it was first found. The use of objects admitting of actual division, such as crayons, pieces of clay, and bits of paper, is recommended. In addition to knowing the parts of a single object, a child also needs to understand the statement that one object is half as long as another. This is quite a different idea of a fraction, and is best understood from an illustration like that on page 19, where one block is seen to be half as large as the second. This is the ratio plan, and is necessary for purposes like those suggested on page 19. It should not, however, be carried to extreme. The third idea relating to fractions, needed by a child at this time, is that of the parts of a group of objects. This is given on page 20. These three ideas should be made the object of a great deal of oral drill, the teacher's problems being better than any examples in a book, because they can easily be related to the children's interests in other work of the school and the home. Children should not take division as a topic in this grade, but should learn that 6 blocks $\div$ 2 = 3 blocks, and that this means the same as one half of 6 blocks equals 3 blocks. There are still a few teachers who talk about "true division," where the dividend and divisor are like quantities, but the mathematical world always recognizes that there are two forms of division, viz.:

6 blocks $\div$ 2 = 3 blocks (sometimes called *partition*);

6 blocks $\div$ 3 blocks = 2 (sometimes called *measuring*).

Children in this grade should not be confused by these distinctions, but teachers should be clear upon this point.

**Forms** (PAGE 21). It is expected that teachers will explain the common terms needed in connection with simple form study, but that they will not ask first-grade children to define them. Hence a picture is introduced showing a child pointing to the horizon, so as to fix the word "horizontal" in the memory and not confuse it with the other long and new word "vertical." The idea of square, oblong, sphere, cylinder, prism, and rectangle can best be given orally. Children learn these ideas best from objects or pictures, just as they learn the meaning of "deer," "elephant," and other words of like nature. Paper cutting and folding is very helpful in the study of the plane figures, and in the work with fractions. The forms given on pages 24 and 25 are merely suggestive of those easily procured by the teacher or made by the pupils.

**Measures** (PAGE 26). The child needs in his first school year to know the inch, foot, and yard. In certain parts of the country he may also hear about the rod and mile, but these measures should not enter into the table to be learned at this time. The measures that are studied should be in use in the class, and the children should come to know them by actually handling them in practical work. For this reason inch cubes should form part of the school equipment, and each child should have a foot rule, even if only a pasteboard one made by himself with the help of the teacher. He also needs to know the cent, nickel, dime, and dollar (as ten dimes). In certain parts of the country he may be given oral problems involving the shilling or bit, but these are local matters that may well be left to the good sense of the teacher. Dealers in school supplies can furnish toy money, although a little real money, with the

children gathered about a table and playing "store," is more satisfactory. The child also needs to know the pint and quart in this grade. He may hear of the gallon and barrel, and may even have problems in which these terms are used, but they should not at this time form part of the tables. Here again the actual measures should be used, made of cardboard if necessary. Most arithmetics give pictures of the pint and quart with nothing to show their relative sizes. Children often think of them as larger than they really are. The picture on page 30 is a constant reminder of their comparative sizes, and is suggestive of their use in the schoolroom. In weight, a child in this grade needs to know only the pound, with its half and fourth. The word "ounce" may be used, but its exact meaning belongs rather to Grade II.

# CHAPTER II

## THE SECOND SCHOOL YEAR

### The Course of Study

**The leading mathematical features.** The addition tables are completed in this grade, and the multiplication tables are learned to $10 \times 5$.

**Number space.** 1 to 1000 both for counting and for writing. Roman numerals to XII.

**Counting.** Count by 1's, 2's, 3's, 4's, and 5's, to 10 times each of these numbers, as a basis for the multiplication table. Also count by 2's, beginning with 1 and continuing to 11, as an exercise in the addition table; and similarly by 3's (beginning with 1 and 2), 4's (beginning with 1, 2, and 3), and 5's (beginning with 1, 2, 3, and 4), extending the counting as far as necessary for the tables.

**Addition.** The addition tables are completed, including all the sums of two one-figure numbers. The 45 combinations ($1+1, 1+2, \ldots, 1+9$; $2+2, 2+3, \ldots, 2+9$; $3+3, 3+4, \ldots, 3+9$, and so on to $9+9$) are to be thoroughly mastered. Numbers of two and three orders; not more than five or six in a column should be used.

**Subtraction.** If taken by the addition plan, it is not necessary to learn any special subtraction tables (see page 14 of this handbook). Subtraction should include three-figure numbers. Special attention should be given to rapidity of written as well as oral work, and to checking (proving) results.

**Multiplication.** The tables should be learned to $10 \times 5$. Products should be memorized both ways, i.e. $6 \times 4$ and $4 \times 6$.

**Division.** Division should be mastered within the range of the multiplication tables. The fact that $6 \times 4 = 24$ should carry with it three other facts, — $4 \times 6 = 24$, $24 \div 6 = 4$, $24 \div 4 = 6$, — and these facts should be learned with the multiplication tables.

**Fractions.** Teach halves, fourths, and eighths; thirds and sixths; unit fractions within the range of the multiplication tables, as $\frac{1}{6}$ of 24, $\frac{1}{4}$ of 24, and so on. Use objective work with cubes, paper folding, paper cutting, blocks, drawings, and other similar material.

**Tables of denominate numbers.** Review the tables of Grade I, continuing the work as here suggested. Weight: ounce, pound; liquid measure: pint, quart, gallon; dry measure: quart, peck, bushel; time: reading time by the clock, the current dates; the square inch; making change to $1.

**Symbols.** Introduce the signs $\times$ and $\div$. The symbol $\times$ is read "times" or "multiplied by"; in business the former is almost universal. $2 \times \$3$ and $\$3 \times 2$ are both recognized forms, but, as already stated, the modern tendency favors the former, the multiplier being written, as it is read, before the multiplicand. It should be remembered that the forms which children need to visualize for practical work are not those in which these symbols appear, but those shown in the annexed examples; hence the written work should emphasize the latter.

$$\begin{array}{r} \$3 \\ 2 \\ \hline \$6 \end{array} \qquad \begin{array}{r} 3\overline{)12} \\ 4 \end{array}$$

**Objects.** Use objects in developing number relations, as may be necessary, but discard them as soon as the relations are established. The number facts must be memorized, and objects may become harmful if used too often.

**Nature of the problems.** In every grade the problems should be related to the children's needs and interests. Oral work should correlate with other school subjects, with games, and with the home life, so far as this is entirely natural. The written work should not neglect abstract computation, always performed with a time limit so as to encourage rapidity. Work should be checked (proved) to encourage accuracy. In general, problems in this grade should involve but one operation.

**Text-book.** Smith's *Primary Arithmetic* may be introduced at the middle of the year, or it may be postponed until the beginning of Grade III, depending on the advancement of the class. The references in this chapter are to Chapter II of the *Primary Arithmetic.*

## Details of the First Half of the Second Year's Work

Chapter II represents the second year's work as laid down in the best modern courses of study. It is divided into two parts, the first representing the review of the first half year, supposing the book to be placed in the children's hands in the middle of the year. If a book is not used until the third year, then Chapters I and II should be reviewed. The work of the first half year includes the operations with numbers to 100, and the second half year extends this to 1000, elaborating the tables of measures.

**Counting** (Page 32). The work is introduced by counting by 10's. As a preparation for this work the children learned to count by 2's and 3's, as set forth on page 15. Teachers will find it advantageous to write on the board the triangle of 10's, like that of 2's on page 15, carrying it to ten 10's. The combination of 10's and units should be

made as on page 34. The splints are helpful for a little time, but should soon be discarded. Objective work is like a crutch, which is helpful until the weak limb becomes strong enough to support the body, and harmful thereafter.

**Addition.** The complete mastery of the table on page 35 is absolutely essential. There must be a development of the table by counting objects, but thereafter it must be so thoroughly memorized as to have the numbers suggest the sum at once, mechanically. The teacher must spend the time necessary to accomplish this, using such devices as number cards (see page 15 of this handbook), and referring both to objects and to mere numbers (abstract numbers). One of the greatest tests of the teacher's ability is that she shall hold the interest of the children while firmly fixing these number facts in their minds. At the end of the first half of Grade II a child should instantly recognize any one of these combinations, and the drill should thereafter be so continued as to prevent any counting in stating the result of any number combination in this elementary addition. Page 36 gives specimens of the large amount of oral drill needed in this grade. Work like $4 + 2$ should lead at once to $14 + 2$, $24 + 2$, $54 + 2$, $84 + 2$, $34 + 2$, and so on. Addition involving two-figure numbers should not, at this time, involve addends where the sum in units' column exceeds 10. It should be remembered that the ideal which we seek in addition is the ability to read a column as we read a word; to look at this column and think 17 without first finding the sum of 6 and 7, and then adding 4. This ideal, however, is not attainable for long columns. Children should not name the addends; they should think, in this case, "13, 17," i.e. the 6 and 7 should suggest 13, and the 13 and 4 should suggest 17.

$$\begin{array}{r} 4 \\ 7 \\ \underline{6} \end{array}$$

**Subtraction** (PAGE 41). As stated on page 15 of this handbook, children should not be confused by several methods. If the "making change" plan is adopted, the child should think, on looking at these figures, "3 and 6 are 9," at the same time writing the 6. The examples lead from $9 - 3$ to $90 - 30$, $99 - 33$, and then to the more general cases like $96 - 32$. No cases requiring the so-called "borrowing" should be given at this time.

$$\begin{array}{r} 9 \\ \underline{3} \end{array}$$

**Multiplication and division** (PAGES 45–46). These are merely touched upon in a very simple way at this time. The emphasis in this half grade should still be upon addition and subtraction. The number table on page 48 is valuable for drill work, and should be copied on a large sheet of paper or on the board. It allows for exercise in counting, adding, subtracting, and so on.

**Roman numerals.** The only use we now have for Roman numerals is in reading the time, in numbering chapters or other divisions, and in reading dates. This last is relatively unimportant at present, and hence the Roman numerals are taught only to XII in this half year. Little attention should be given to the subject in any grade, the work being confined chiefly to reading the numbers.

## DETAILS OF THE SECOND HALF OF THE SECOND YEAR'S WORK

**Reading and writing numbers** (PAGE 51). The number space is now extended to 1000, although the operations are still with the smaller numbers. We generally need to count and to read numbers much beyond the limits of our operations. The use of the splints on page 51 should not take much time. The children have already learned to

count by 10's and they readily take up counting by 100's. In reading a number like 123 it is better to say "one hundred twenty-three" than "one hundred *and* twenty-three," although the latter is perfectly good English. The reason is that a little later the child will find the "and" confusing except when he crosses a decimal point or passes from an integer to a common fraction. For example, just as $2\frac{1}{2}$ is read "two and a half," so 100.023 is read "one hundred and twenty-three thousandths," while 0.123 is read "one hundred twenty-three thousandths." Teachers should also lead children to avoid the vulgar inaccuracy of "aught" for "naught." "Aught" means *anything*, while "naught" (i.e. "not aught") means *not anything*, i.e. *nothing*.

**Measures** (PAGE 55). The new measures now needed are the ounce, and the quart, gallon, peck, and bushel. These should all be introduced with the actual measures at hand. Failing this, the illustration on page 60 will be very helpful. The table of time (page 63) is also given in this grade, as is the introduction to square measure (page 64).

**Counting** (PAGE 66). Children like to count. The rhythm is pleasing, and counting enters into many of their games. They like to display their powers in this respect. It is perfectly natural that this should be so, for the primitive arithmetic was little more than counting. It is desirable to utilize this number instinct, and this is easily done, as suggested on pages 66–78. If we count by 2's from 2 to 20, we really give a multiplication table of 2's, besides the addition of 2's with respect to even numbers. If we count by 2's from 1 to 11, we give the addition table of 2's with respect to odd numbers. There is no particular object in counting by 2's beyond these limits, and the time necessary for further counting can be employed better on work with other numbers.

**The multiplication table** (PAGES 70–78). In the second year it is advisable to carry the table only to $10 \times 5$. The table is best developed by means of counting, objects being used only as necessary. It should be mastered in its various forms, as set forth on pages 77–78. That is, the child should know and relate these facts:

$$6 \times 3 = 18, \quad 3 \times 6 = 18, \quad 18 \div 3 = 6, \quad 18 \div 6 = 3.$$

These facts are best learned in the first instance from counting; then in table form, as on page 77; and finally in isolated form, the teacher calling for the results of $6 \times 3$, $5 \times 2$, $4 \times 7$, and so on in miscellaneous order.

**Addition** (PAGE 79). This is now taken up with the so-called "carrying" feature. The explanation on page 79 is sufficient, and children should always be taught to check their work. The most essential feature of the computations of a good calculator is the checking of his work at every step. The long form of addition, writing the results separately for each column, should be given first. After that is entirely familiar, the short form on page 80 should be given. There is no danger, with a good teacher, that children will keep to the first form, because if they are timed in their work, as should always be the case to avoid slothful habits, they will naturally come to use the shortest method possible. In the same way with a three-figure number (page 81), it is well to separate the addends as shown, as a basis for the explanation below, but children will not take the long form if they are encouraged to work rapidly.

**Subtraction** (PAGE 84). As already stated on page 14, there are several methods of subtraction, and a school should adopt one and adhere to it throughout the grades so as to avoid confusing the children. The best method is the one that is the most quickly performed, independent

of the matter of explanation. Any method can be explained or developed, and children should not be required to repeat an explanation. The great thing for them is to see that the method is a valid one, and then to devote their energies to problem solving. Hence it is well that they separate the numbers into parts, as on page 85, in a half-dozen cases, and then that they subtract a great many times so as to fix the habit. A satisfactory method (but by no means the only one) is the one first indicated on page 85:

$$\begin{array}{r} 523 = 400 + 110 + 13 \\ \underline{154} = \underline{100 + \ \ 50 + \ \ 4} \\ 369 = 300 + \ \ 60 + \ \ 9 \end{array}$$

4 and 9 are 13, 5 and 6 are 11, 1 and 3 are 4. The teacher can easily develop this, using bundles of splints if necessary, but not otherwise. For rapidity, however, the best method is that which increases the second and third figures of the subtrahend by 1. This is easily explained by first showing that the result is not changed if the minuend and subtrahend are both increased by the same number, as by 10, 100, etc.

$$\begin{array}{lr} 523 = 500 + 20 + 3 & 500 + 120 + 13 \\ 154 = 100 + 50 + 4 & \underline{200 + \ \ 60 + \ \ 4} \\ & 300 + \ \ 60 + \ \ 9 \end{array}$$

Here both minuend and subtrahend have been increased by $100 + 10$.

**Multiplication** (PAGE 87). Following the principle already suggested, that of giving the complete operation first and then the contracted form, three plans are given on page 87, of which the third is the only one for the actual use of children. Like all such work, this should be developed on the board by questioning the children, and the

method should be understood, but a pupil should not be expected to repeat the explanation in this grade; he should be set to work at the actual multiplications, and encouraged to work rapidly. Seat work without a time limit leads to habits of dawdling.

**Division** (PAGE 90). Here again is illustrated the principle that the complete form should precede the short one. A child much more easily grasps the idea of $36 \div 3$, if he uses it in the first of these forms before he sees it in the second:

$$\begin{array}{r} 3\underline{)30+6} \\ 10+2 \end{array} \qquad\qquad \begin{array}{r} 3\underline{)36} \\ 12 \end{array}$$

Some teachers, observing that the quotient is written above the dividend in long division, teach this in short division, writing the numbers as here shown. It is not recommended, however, for several reasons, the chief of which is that it is not the form actually used in business. Children are never troubled, with a good teacher, about the reason for writing the quotient above in long division, and the school cannot afford to be teaching forms that must be unlearned in business. There is the added reason that we often wish to continue the division, as in the annexed example, in which case it is easier to have the quotients run down the page.

$$\begin{array}{r} 12 \\ 3\overline{)36} \end{array}$$

$$\begin{array}{r} 2\underline{)1728} \\ 2\underline{)\ \ 864} \\ 2\underline{)\ \ 432} \\ 2\underline{)\ \ 216} \\ 108 \end{array}$$

**Fractions** (PAGE 91). To the fractions already studied are now added $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{8}$. The arrangement of blocks on page 91 will be found very helpful in all fraction work. This scheme allows us to combine all of the three important notions of a fraction, — (1) a part of one thing (as a half of a column), (2) half as large (as where one column is

half as high as another), and (3) half of a group (as half of 8 cubes).

**Concluding work for the second school year.** The teacher should devote the close of the year to satisfying herself that the pupils know thoroughly the addition table and its application in subtraction; that they know thoroughly the multiplication table through $10 \times 5$, in its various forms, with the inverse division table; that they know thoroughly the facts of these tables if asked out of their regular order; and that they can perform the fundamental operations involving these tables, with numbers below 1000, quickly and accurately. While the problems should be made attractive and genuine, as in this book, it must be remembered that interest is valuable only as an incentive to hard work. It is not enough that a child knows a reason for $27 + 8 = 35$, or that he can count from 27 to 35, or that he can illustrate a sum with inch cubes; these things are valuable and interesting, but they do not constitute the great end in view. The great thing is that the child should think 35 the instant he sees these figures and is told to add, and that he should think 19 the instant he is told to subtract. After the long summer vacation (if the class is promoted at this season) the children will have forgotten enough to discourage the teacher in the third grade at best, and we need to see to it that her task is made no harder than necessary, and that she has no just cause for criticism. After all, there is no panacea for the necessity of a great amount of drill, and rapidity and accuracy in the work of the second grade mean the fixing of a habit that will make all subsequent work stronger.

$$\begin{array}{r} 27 \\ \underline{8} \end{array}$$

# CHAPTER III

## THE THIRD SCHOOL YEAR

### THE COURSE OF STUDY

**The leading mathematical features.** Rapid written work together with the oral. Multiplication as far as two-figure multipliers. Long division begun. The most useful tables of denominate numbers completed.

**Review.** Begin with a rapid and systematic review of the preceding work, so that the status of all the children may be known, and any backward ones may be assisted.

**Number space.** 1 to 10,000. Roman numerals through M. For purposes of counting, the numbers may be extended to 100,000, but 10,000 will suffice for operations.

**Counting.** Continue the counting of Grade II, to include 6's, 7's, 8's, and 9's, as a basis for the multiplication tables and as a review of the addition combinations.

**Addition.** Review the 45 combinations. In the first half year, oral work of the type $36 + 40$. In the second half year, oral work of the type $26 + 42$ and $342 + 24$, where no "carrying" is involved. Written work with numbers of four orders, including dollars and cents. Special attention to accuracy and to checking all work. Reasonable rapidity secured by placing a time limit upon all written work. Long columns of figures to be avoided.

**Subtraction.** As suggested by the work in addition.

**Multiplication.** This, with division, constitutes the special work of the year. In the first half year the multiplication

tables are completed through $10 \times 10$, and are made such an object of drill that the facts are thoroughly memorized not merely in tabular form but when taken at random. In the first half year, multiplication of numbers of three orders by numbers of one order. In the second half year, the multiplier is extended to two-order numbers, including those ending in zero. The multiplicand may represent dollars and cents. Special attention to accuracy, to checks on the operation, and to rapidity, as in addition.

**Division.** Dividing at sight, with remainders. In the first half year, written division of numbers of four orders, the divisor being of one order; no remainders. In the second half year, short division by 20, 30, etc., and long division with divisors whose unit figure is 1 or 2.

**Fractions.** Halves, thirds, fourths, and fifths, in the first half year. Halves reduced to fourths and to sixths, and thirds to sixths, in the second half year. Oral addition and subtraction of such fractions. In the second half year, addition and subtraction of mixed numbers containing these fractions, the fraction in the minuend being larger than the one in the subtrahend. Fractional parts of integers of three orders, which are multiples of the denominators. Objective work when necessary.

**Denominate numbers.** Review the tables already studied. Teach how to write dollars and cents. Square inch, square foot, and square yard, with diagrams illustrating areas. Objective work in finding volumes in cubic inches. The ton. The gill added to the table of liquid measure. Table of time completed.

**Nature of the problems.** Problems may now include two operations, as the finding of the cost of five articles when the cost of three is known. Problems should appeal to the child's interest, should be genuine, and should involve the

various operations studied. Oral problems should relate to local conditions, including the other studies of the class.

**Text-book.** Smith's *Primary Arithmetic*, Chapter III, covers the work of this grade, and is conveniently divided for the two parts of the year. It also offers a review of the work of Grades I and II.

### Details of the First Half of the Third Year's Work

**Review.** There is almost always a complaint after the first school year, that children are not well prepared for the work of the grade they are entering. This complaint is most marked in such studies as arithmetic, spelling, and writing, because defects are here more easily seen. It is true that the encroachment of more modern topics upon the time formerly given to studies like arithmetic and spelling, and to practice in penmanship, has prevented that elaborate drill which was formerly allowed. Nevertheless we have time enough to allow for the essentials in every subject now in the curriculum, provided our teaching is good. The complaint of insufficient preparation cannot, therefore, be dismissed on the ground that poor preparation is necessary. As a matter of fact, the difficulty is usually with the teacher who makes a complaint and who thus seeks to excuse her own shortcomings. Where, as is frequently the case, the promotion from grade to grade is between spring and autumn, children have two months in which they have practically no exercise in number work. They learn quickly and they forget quickly at this age, the brain being very plastic in early childhood. It is therefore entirely natural that children who were bright and accurate in their arithmetic in the second school year should enter

the third grade with a very hazy notion of the whole subject as previously studied. The first duty, therefore, of every teacher is to spend two or three weeks or longer in a review of the essentials of the preceding grade. Indeed, one of the tests of the teacher's ability is found in the interest maintained while taking up such a review, and in the results of her efforts to reimpress on the brain the half-obliterated impressions of the preceding year.

**Reading and writing numbers** (PAGE 94). In the third grade the number space is usually limited to 10,000, the actual numbers used in operations being still rather small.

2000 + 300 + 40 + 2

The introduction to the subject is similar to that in the case of numbers to 1000, splints being used if needed, but the class taking up abstract counting immediately after. It is a mistaken idea that children need to visualize objects when they think of numbers. We rarely do so ourselves, a number being to us a name in a series, which name we can, when we need to, relate to a group of objects. The oral exercise on page 94 will suggest a large amount of drill in reading and writing numbers from 1 to 10,000. The most obvious use that the child has, at this time, for numbers between 1000 and 10,000 is in connection with dates, and hence this feature is emphasized on page 95. The Roman numbers are usually learned to C at this time, this limit being sufficient for chapter numbers. Again it is suggested that the child's only use for these numerals

being in reading, his exercise in writing them should be merely to assist him in this work.

**The decimal point** (PAGE 96). Children now need to learn how to write dollars and cents, although they are not ready to understand the theory of decimal fractions. Hence the symbols are given here, and the children write $12.50 and $175.05, learning the correct name of the decimal point. Just as they have added 426 and 229, so they add $4.26 and $2.29, and they will not have any difficulty in the matter unless the teacher confuses them by over-explaining.

**Forms** (PAGE 100). Such simple forms as certain of the triangles are now introduced with their correct names. Schools that do work in paper cutting or paper folding will take advantage of this fact at this time. The actual hand work required on page 103 is helpful both on the manual training side and in making the subject of cubic measure (page 104) seem more real and interesting.

**Square and cubic measure** (PAGES 101, 104). With the help of a figure like this, and of the explanation given on pages 101, 102, and 104, children will have no difficulty with this simple introduction unless they are confused by their teachers; but if a teacher talks about

3 in. times 3 in. equals 9 sq. in.,

instead of

3 times 3 sq. in. equals 9 sq. in.,

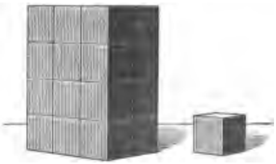
the children will at once acquire loose habits of thought and expression, and there will be trouble ever after. The best form for written work, because it is true and has the merit of brevity, is probably

$$3 \times 3 \text{ sq. in.} = 9 \text{ sq. in.},$$

the multiplier being written first (see page 15 of this handbook), as it is read, and the multiplicand expressing the number of square units in a row. The form

$$3 \times 3 \times 1 \text{ sq. in.} = 9 \text{ sq. in.}$$

is correct, but sometimes the children do not grasp the refined idea, here expressed, of emphasizing the unit. For the development of this work square inches may be cut from cardboard or drawings may be made.



In the same way

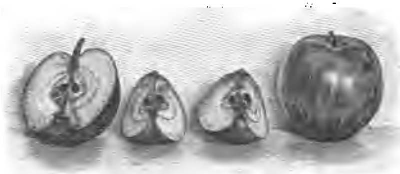
$$4 \times 2 \times 3 \text{ cu. in.} = 24 \text{ cu. in.}$$

or

$$4 \times 3 \times 2 \text{ cu. in.} = 24 \text{ cu. in.}$$

is a good form for the work in cubic measure. For the development of this work teachers should use inch cubes, because a drawing cannot be understood as in the case of square measure. Inch cubes can be procured from any dealer in school supplies, and are usually found in schools. Enough cubes can easily be made from cardboard, however (see page 103), to answer the purposes.

**Fractions** (PAGE 106). The work in fractions must still be largely oral. The subject is too new and offers



too many difficulties to children to make it desirable to give much written work at present. Written work is

reserved for the latter half of this grade. Teachers must of necessity resort to objective work at this stage, and the traditional apple is always useful for this purpose. They will also find the use of the columns, as here shown, very advantageous. The child can here see, without any unnecessary difficulty, that $\frac{1}{4}$ of 12 is 3, that $\frac{1}{2}$ of 12 is two 3's, or 6, and so on. In connection with this column arrangement the array suggested on page 69, and here reproduced, is very helpful.

| | | | |
|---|---|---|---|
| 3 | 7 | 10 | 25 |
| 3 | 7 | 10 | 25 |
| 3 | 7 | 10 | 25 |
| 3 | 7 | 10 | 25 |

| | | | | |
|---|---|---|---|---|
| | | | | 2 |
| | | | 2 | 2 |
| | | 2 | 2 | 2 |
| | 2 | 2 | 2 | 2 |
| 2 | 2 | 2 | 2 | 2 |

**Addition** (Page 108). Teachers are strongly urged to introduce the full form of each operation at first and the abridged form later. It is sometimes objected that the full form (the sums of the columns appearing separately) will keep children away from the abridged form (the sums of the columns not written separately). This, however, is not a valid objection. As a matter of fact, business men themselves usually find it better to write the sums of the columns separately, so that if a child forms this habit for addition it is not a serious matter. It is true, however, that a child usually enjoys taking contracted forms as soon as he can do so with safety.

It is suggested that children should group by tens whenever possible. It should be understood, however, that a good teacher will see to it that children know the other combinations as well as the combinations of tens. For example, a child should be able to group by eights, or to see that 8 and 9 are 17, and that 9 more make 26, in the example given. It is a fact, however, that we can add two tens with less mental difficulty than an eight and a nine; and hence the suggestion of grouping by tens is a valuable

one, and one which all computers follow whenever they can easily do so.

**Subtraction** (PAGE 109). Teachers should read the suggestions concerning subtraction already made on pages 84 and 85 of the *Primary Arithmetic* and on page 25 of this handbook. There is no new principle involved at this point. The separated terms in the middle of page 109, and here repeated, show the work as it may be presented

| *This shows all the work:* | *But we write only this:* |
|---|---|
| $1632 = 1500 + 120 + 12$ | $1632$ |
| $756 = \underline{\ 700 + \ 50 + \ 6\ }$ | $\underline{756}$ |
| $800 + \ 70 + \ 6 = 876$ | $876$ |

by the teacher on the blackboard. A better but not so popular form (see page 26 of this handbook) is the following:

$$\begin{array}{rr} 1632 & 1600 + 130 + 12 \\ 756 & \underline{800 + \ 60 + \ 6} \\ & 800 + \ 70 + \ 6 = 876 \end{array}$$

Children should not be expected to separate the terms, but teachers will find this the best method of explaining the process. Objects such as bundles of splints are rarely needed as late as this.

In the drill work suggested on pages 110 and 111 teachers are urged to place a time limit upon the work of the pupils. If a page of problems is given, a pupil is quite liable to take an undue amount of time, and this establishes habits of slothfulness. Teachers will soon find the amount of time necessary for the solution of ten problems, and the children should then be limited to this length of time.

**Counting by different numbers** (PAGE 112). It is now generally recognized by teachers that it is a good plan to

introduce the multiplication table as an exercise in counting by various numbers. This plan has already been suggested in this handbook on page 13. It is not wise, however, to spend a great amount of time on counting that does not lead to anything, and it will be noticed that all the counting here suggested leads directly to the multiplication and addition tables. The column arrangement on page 112 will be found very helpful, and the triangles should extend as far as 10; for example, to ten 6's, or to twelve 6's, if the multiplication table is carried that far. The desirability of learning both forms of a product, as $2 \times 6$ and $6 \times 2$, is apparent. They should be learned together and the drill should be given on both forms. The desirability of coupling this with the division forms is also apparent. The question is sometimes raised as to the desirability of having children learn tables, the feeling being that they have to recite them before they are able to find what fact they are after. This feeling, however, has no standing with good teachers. They recognize the value of the table, and at the same time they give a great deal of drill upon the miscellaneous facts of it. When this is done rapidly children come to know the facts independent of their position in the tables themselves.

**Multiplication** (PAGE 121). Following the general principle that the complete form should first be given by the teacher in developing a new process, and then the practical abridged form, the work on page 121 is arranged so as to bring out first the reason and then the practical form. It should always be borne in mind that the child should not be asked to accept a process without first understanding it, but having understood the reason for it he should come to perform the operation mechanically. It is not necessary or desirable that a book should give an undue number of

problems of the kind stated on page 121. There is a great interest on the part of pupils at this age in solving problems that are not in the book. Since these problems are so simple that a teacher can dictate them without any difficulty, it is suggested that numerous written drills be given with a time limit upon each, the problems being dictated or written upon the board, and the children's answers compared at the end of five or ten minutes. Neatness of work should be insisted upon, it being universally recognized that a good computer puts his work in neat form.

$$\begin{array}{rl}
438 & \\
\underline{6} & \\
48 = 6 \text{ times } & 8 \\
180 = 6 \quad \text{"} & 30 \\
\underline{2400} = 6 \quad \text{"} & \underline{400} \\
2628 = 6 \quad \text{"} & 438
\end{array}$$

*Write only this:*

$$\begin{array}{r}
438 \\
\underline{6} \\
2628
\end{array}$$

**Division** (Page 123). The question is sometimes asked whether it is better to teach long division before short division. The answer is that there is no essential difference in theory between the two processes. If teachers will separate the dividend as suggested on page 123, short division involves no principle that is not in long division, and vice versa. The separating of the dividend is merely a means to the understanding of the process, and teachers should write it upon the board and use it enough for the pupils to understand the reason involved. The children should not be required to make this separation, but should at once get to the practical business method of dividing. Here again the necessity for a considerable amount of written drill work is apparent, and a time limit should be set upon all such work. The remainder is introduced on page 124, and this being done, teachers can easily dictate any number of examples without regard to exact divisibility. It is then an interesting feature of the work to

$$\begin{array}{r}
6\overline{)480 + 42} \\
80 + \phantom{4}7 \\
= 87
\end{array}$$

compare the answers of the children. As an oral exercise the work on page 124 is exceedingly valuable. Teachers should write columns of figures like those in Ex. 1, placing the divisor on one side and asking for the quotient and remainders. The divisor may then be erased and another written in its place. Sets of examples like those on page 126 are introduced because they appeal to the heroic instincts that are so prominent with children at this age. Other sets of examples, like those on page 122 and page 127, appeal to the children's love of nature and touch the business interests of various sections of the country.

## Details of the Second Half of the Third Year's Work

**Review.** If there has been a vacation between the two half years, teachers should give a thorough review, as suggested in this handbook on page 31. It is not customary to introduce any more extended counting in the second half of the third year. Usually the Roman numerals are increased a little, most courses of study suggesting that they extend to M.

**Addition** (Page 131). It is recommended that a large amount of oral drill, like that suggested on page 131, be required. It is a great mistake to let children feel that they must take a pencil and paper to add numbers of two figures. It will be found later that oral addition is required even when the sum of the units' figures is greater than ten. Children should therefore become accustomed to handling this work easily at this time.

**Subtraction** (Page 123). See the suggestion already made on page 36 of this handbook. The separating of the

numbers into parts must always be understood to be a device to lead the children to understand the process more fully. They should come to work mechanically, however, just as every good computer does. The suggestion made on page 134, that pupils should repeat the results only, without repeating the numbers, will be appreciated by all successful teachers. When a child repeats the numbers unnecessarily, it usually means that he wishes to gain the time to count on his fingers or to repeat the table to himself. The dogma of "the complete answer" has little place in oral arithmetic.

The suggestion that pupils should check their results is one that no teacher should forget. The first requisite in the making of a good computer is the learning how to check the work at every step. If the pupil has performed an addition, he should add in the opposite direction. If he has performed a subtraction, he should check by addition. If the subtraction has been made by the modern addition plan, it should be checked by adding in the opposite direction.

**Multiplication** (PAGE 136). The device given on this page of placing numbers about a circle, with the multiplier or divisor inside, is an interesting one for occasional use. It should always be borne in mind, however, that such a device is valuable only as an interesting change, and it should not be carried to any dangerous extreme. Children should visualize the numbers in the form in which they are going to use them, the multiplier being placed under the multiplicand and the divisor to the left of the dividend.

It will be noticed that the way is here being prepared for multiplying decimal fractions. Children should not, however, be troubled with these fractions at the present time. All they need of such work is included in the use of

dollars and cents. The decimal fraction as such is not taken up until page 242 of the *Primary Arithmetic.*

**Division** (PAGE 142). The question is often raised as to the best time for the introduction of the two kinds of division (page 142). This must be left to the judgment of the teacher. It depends upon the maturity of the class. In certain parts of the country and in certain parts of a city children will be quite able to understand the two kinds of division at this time. In other parts of the country and in other parts of the same city, however, the matter may preferably be postponed. It must, however, be presented before long, and when presented the teacher will naturally use objective work wherever it is necessary.

As in the case of multiplication above mentioned, the division on page 143 forms an introduction to the division of decimal fractions, which is taken up considerably later. Children do not need the division of decimals at the present time, but it is necessary that they should understand the division of United States money. The oral exercise at the top of the page introduces, as usual, the theory in the middle of the page. The same thing is true on page 146, where the oral exercises introduce the pupil to the theory, and where the complete work, as set forth by the teacher, is followed by the abridged work which the pupils are to use.

**Fractions** (PAGE 152). It is still necessary to introduce new work in fractions by the help of objects. The subject should be made as real as possible through the use of paper cutting, paper folding, and measurements. The rapid drill work suggested on page 154 may safely be given very frequently, it not being necessary that a pupil should visualize a definite form here, as is the case in the multiplication and division of integers. When children are taking a third

of any number it is usually done orally, and therefore these circle forms are entirely safe. The suggestion on page 158, that the objects should be discarded as soon as they cease to be necessary, is one which teachers would do well to follow. One of the great mistakes of modern teaching has been the use of objects after they cease to be necessary, and the insisting upon children becoming too familiar with them and not familiar enough with the practical computing.

**Measures** (PAGE 161). Three new tables are introduced at this time, and as far as possible children should visualize the units involved; that is, the number of square inches in a square foot should be seen from an actual drawing upon the board, and the same is true for the number of square feet in a square yard. Teachers can easily show to the pupils a ton of hay or a ton of coal, so that they may form an idea of the approximate size of this weight of certain substances. The actual measures should be used in this grade, as already set forth on page 17 of this handbook. The important basal units of our common system should be familiar to every child.

# CHAPTER IV

## THE FOURTH SCHOOL YEAR

### THE COURSE OF STUDY

**The leading mathematical features.** Multiplication and division with three-figure multipliers and divisors. The common business fractions.

**Review.** Begin with a rapid and systematic review of the preceding work. The complaint that children are not prepared for their grade is usually due to a failure in beginning the year's work by refreshing the children's memories.

**Number space.** In the first half year, 1 to 100,000. In the second half year, to billions, but with the operations confined to the smaller numbers of common business.

**Counting.** Count by 11's and 12's as a basis for completing the multiplication table to $12 \times 12$ if desired, and as an exercise in addition.

**Addition and subtraction.** Rapid oral work with numbers like $25 + 7$, $25 + 17$, $25 + 37$, $62 - 25$. Much rapid oral work without a book, the numbers being either spoken by the teacher or written one above the other on the board. In the written work absolute accuracy secured by the constant use of checks, and rapidity secured by daily drills to see how many examples can be solved in five minutes.

**Multiplication and division.** Multiplication tables may be extended to $12 \times 12$ if desired. Oral multiplication of any two numbers whose product is less than 50, as $3 \times 16$,

$2 \times 19$. Multipliers and divisors of three orders, in written work. Accuracy and rapidity secured as in addition and subtraction.

**Fractions.** Special attention here, as in all later grades, to the common business fractions from $\frac{1}{2}$ to $\frac{1}{8}$. Recognition of 50¢ as $\$\frac{1}{2}$, and 25¢ as $\$\frac{1}{4}$, in finding the cost of articles, as in the cost of 24 arithmetics at 50¢ each, or the cost of 12 yards of cloth at $1.25 a yard. Addition and subtraction of fractions where the least common denominator can be found by inspection. Easy reductions. Easy multiplications involving integers and fractions, two fractions, or an integer and a mixed number.

**Decimal fractions.** A brief introduction based upon common fractions and the table of United States money.

**Denominate numbers.** Constant review of tables, with practical applications. Tables of long measure and cubic measure completed.

**Measurements.** Systematic application of the tables to practical measurements. Areas of floors, of school grounds, and of various plots of land. Measuring of perimeters and the study of the cost of fencing. Diagrams drawn to scale.

**Problems.** As in the preceding grades, the problems should relate to the interests of the child and his immediate needs. The heroic element is now strong and problems may recognize this fact. The child is now making purchases, and this fact should also determine the nature of many problems. Simple bills should be introduced. Problems may now require two or three operations, and some simple analysis may be demanded.

**Final review.** The year's work should close with a brief and systematic review, to make sure that accuracy and rapidity have been secured in the fundamental operations,

both in written and oral work. If a complaint comes from Grade V that the children are ill prepared, the teacher in Grade IV should be assured, after this review, that it is because the next year's work has not been properly begun.

**Text-book.** Smith's *Primary Arithmetic*, Chapter IV, covers the work of this grade and is conveniently divided for the two parts of the year.

## Details of the First Half of the Fourth Year's Work

Chapter IV contains the fourth year's work as laid down in the best modern courses of study. It is divided as usual into two parts, the first covering the work of the first half year and the second that of the second half year.

**Counting reviewed** (Page 171). It should be borne in mind that the object of counting is to develop the tables of addition and multiplication. This has now been accomplished, and therefore the reason for continuing the exercise is merely for drill upon those tables.

**The fundamental operations reviewed** (Page 174). Children should have plenty of exercise in such oral work as that given on pages 174, 176, and 179. It is a great mistake to allow children to take much time in adding numbers like 45 and 69. This work is easily done orally, and rapidity should be insisted upon. In adding it is often easier to begin at the left; for example, the sum of 67 and 58 may be found by thinking that 67 plus 50 equals 117, and that this plus 8 equals 125. In the same way it is often easier to begin at the left in oral subtraction. The work of multiplying by a three-figure multiplier (page 180) offers no new difficulties. It is well to give the complete form first by way of explanation, but to require

the children to give only the short form at the bottom of page 180. It will be noticed that the work in long division (page 182) was introduced on pages 146–148 by taking divisors whose units' figure was 0, 1, or 2. The reason for this plan is that children can thereby tell without any difficulty the various figures of the quotient, and are not liable to get a figure that is too large. Having had sufficient practice in this work, they are now (page 182) ready to undertake division where the units' figure is larger than 2. It will be noticed (as on page 184) that the oral work constantly leads to the theory in the middle of the page, and that abundant drill work is given at the bottom of the page. Since this work in long division is one of the leading features of this half year, rapidity of work and accuracy of result should be required. Children should not enter upon the work of the second half year without being thoroughly familiar with this subject.

**Fractions** (PAGE 190). The difficulty of fractions is now slightly increased, and although some resort to objective work is still necessary, as shown on page 192, it must always be remembered that it is well to leave the objective work as soon as possible. The explanations given on the middle of the various pages (as on page 194) are usually sufficient for the understanding of the theory. These explanations are necessarily condensed, and teachers may need to amplify the work as the nature of the class demands.

**Denominate numbers** (PAGE 203). It is a fact that we have very little use for compound numbers involving more than two denominations. The best type of compound numbers is given on page 210. Teachers are advised not to introduce numbers with three or four denominations, such as were formerly found in arithmetics. A large number of

practical examples is better than a small number of very long and impractical ones. In explaining the process of finding the areas of triangles and parallelograms (page 205) teachers are advised to use paper cutting.

The form for finding volumes given on page 207, or the ones on page 34 of this handbook, should be insisted upon. If teachers allow incorrect forms, such as 3 ft. $\times$ 3 ft. $\times$ 3 ft. $=$ 27 cu. ft., they must expect inaccuracy on the part of pupils in other matters. The best plan for leading children to understand the reason involved in this process of finding volumes is to give them inch cubes from which to build up solids, as shown on page 207.

In the work with denominate numbers resist all temptation to inaccuracy of form. It is inexcusable for a teacher to allow forms like these:

$$60 \text{ in.} \div 12 = 5 \text{ ft.},$$
$$60 \div 12 = 5 \text{ ft.},$$
$$60 \text{ in.} \div 12 \text{ in.} = 5 \text{ ft.}$$

If we wish to reduce 60 in. to feet, we have three possible correct forms:

$$60 \times \tfrac{1}{12} \text{ ft.} = 5 \text{ ft.},$$
$$60 \text{ in.} \div 12 \text{ in.} = 5, \text{ the } \textit{number} \text{ of feet},$$
$$60 \div 12 = 5, \text{ the } \textit{number} \text{ of feet}.$$

The explanation of the first is: "since 1 in. $= \frac{1}{12}$ ft., 60 in. $= 60 \times \frac{1}{12}$ ft., or 5 ft." The explanation of the second is: "since 12 in. $=$ 1 ft., 60 in. $=$ as many times 1 ft. as 12 in. is contained in 60 in., or 5 times; and $5 \times 1$ ft. $=$ 5 ft." But if slovenly forms are allowed here, teachers must expect that the work in the fifth grade will be inaccurate and that there will be a just cause of complaint that the seeds of error have been sown in the fourth grade.

## Details of the Second Half of the Fourth Year's Work

**Counting** (Page 213). The number names are now given as high as a billion. Children, however, have little use for numbers as large as these. Indeed, it is not settled in the English language whether a billion means a thousand millions or a million millions. The majority of English-speaking people do not follow the American custom, but take a billion to mean what we call a trillion.

**The fundamental operations** (Page 215). Children should see that the addition of compound numbers (page 215) involves no new principle. The columns are added separately, as in ordinary cases. It is not necessary at present to explain the fact that these numbers are written on a varying scale, and that this is the only difference in the method of operating with them. This is the case, however, and some slight suggestion to this effect may be helpful.

It need hardly be said that a very great amount of written drill should accompany the oral work at this time. There is a tendency on the part of some schools to continue the oral work to the exclusion of the written work. This is a dangerous mistake, for in dealing with three-figure numbers the work is almost always written, and pupils should acquire facility in handling such work. Rapid written reviews of important pages like 173–189 are valuable. If it is necessary to review long division occasionally in this half year, pages 182–189 offer sufficient material, rapidity being encouraged. Some teachers advocate, in the reviews, the abridging of such work as that on page 182 by omitting the partial products, as here shown. Here a good computer will say, "3 × 3 are 9, and 1 are 10," writing the 1; "3 × 7

$$\begin{array}{r} .37 \\ 73\overline{)2701} \\ 511 \end{array}$$

are 21, and 1 is 22, and 5 is 27," writing the 5. This gives 51 tens, making 511 as the complete remainder. In the same way, $7 \times 3 = 21$, and 0 is 21; $7 \times 7 = 49$, and 2 are 51, and 0 is 51. Therefore there is no remainder. It is possible to simplify this explanation for children, but it is not advisable to attempt it. Such abbreviations are too taxing, and may well be left for professional computers.

**Fractions** (PAGE 223). The field for fractions is now again enlarged, particularly to include the case of least common denominator (page 228). In examples suited for primary work this denominator can always be obtained by factoring. Indeed, the old custom of having fractions with very long numerators and denominators is passing away. Such fractions are not commonly used in business and should be introduced only as occasional drill work. Children may reasonably enjoy the contest with a large and unusual fraction at times. It is, however, better to give more drill with the fractions commonly used, and less with those that are not usually met in business. The presence, in the old-style arithmetics, of long fractions like $\frac{1373}{4211}$ is explained when we consider that decimal fractions were not invented until about 1600, and that these long fractions were once very common. We now reduce them to decimal fractions, carrying them to two or three places as the occasion may require. The conservatism of the schools kept these large fractions in the books long after their utility was gone, but they are now rapidly disappearing. The idea that they involved any "mental discipline" not to be derived from more modern examples is not well founded.

Some teachers advise using such forms as the annexed one in adding fractions. Here the common denominator is written above the numerators 9 and 10. Such devices

are not, however, recommended. It is much better to use the complete forms, as on page 228; for if they are understood these devices are easily picked up if occasion demands. Children at this stage of development are not ready for unusual abbreviations, such as are not commonly used by the business world. The same remark holds true for the subtraction of fractions (page 229).

$$\begin{array}{r|r} & 12 \\ \hline 3\frac{3}{4} & 9 \\ 7\frac{5}{6} & 10 \\ \hline \end{array}$$
$$10 + \tfrac{19}{12} = 11\tfrac{7}{12}$$

Such devices as those on page 233, as in the use of $60 \times \$\frac{1}{4}$ instead of $60 \times 25$ ct., are very valuable. They are used in everyday business, and children should become accustomed to them early and should be drilled on them constantly. It is absurd that a pencil and paper should be needed for an example like finding the cost of 12 articles at \$1.50 each (see page 234). The pupil should say at once, "They will cost \$12 and half as much more, which is \$12 + \$6, or \$18."

The triangular arrays on page 235 should be continued as far as ten figures in the last column. These arrays are very helpful in the teaching of fractions. The child points to a column where the sum is 30, and then to columns showing $\frac{1}{2}$ of this number, $\frac{1}{3}$ of it, $\frac{5}{6}$ of it, and so on.

**Bills and receipts** (PAGE 237). In some parts of the country children study very little arithmetic after finishing the first book. Many of them go into business, leaving school permanently. It is therefore very desirable to have the most common business forms understood. For this reason a little work on bills and receipts is given, limited to the common needs of the household or the store. Neatness should be insisted upon, for slovenly work breeds inaccuracy.

**Decimal fractions** (PAGE 242). There was formerly a dispute among teachers as to whether decimal fractions should

precede common fractions. It is now quite generally agreed that the simpler common fractions should be given very early; that the writing of United States money should be given in the third grade; and that decimal fractions as such should not be given before the latter part of the fourth grade. This is in harmony with the development which the world has undergone. The decimal fraction was not used until three hundred years ago, and was not at all common until about a hundred years ago. The common fraction has, however, been used for many centuries. All that the child needs of decimal fractions before the fourth grade is what he gets in handling United States money. Before he leaves the primary grades, however, he should understand the principles of the decimal fraction. These fractions should be based on the table of United States money as suggested upon page 242. Objective work is not as necessary here as it would have been had these fractions been introduced earlier. The child is already familiar with the decimal point, and the objective illustrations given on pages 242 and 243 will probably suffice.

**Denominate numbers** (PAGE 248). All the tables of denominate numbers that are necessary for the primary grades have now been given, excepting the table of land measures on page 249. An effort should be made to have children visualize these standard measures in the same way that they have visualized the standards in the earlier grades. The note on page 249 should be carefully considered.

**Percentage** (PAGE 258). This work is usually postponed to the fifth grade. It is introduced here for the benefit of schools extending this book into that grade, and for those children who will probably leave school after its completion. The introduction is very simple and covers only those cases that are of great importance. The one thing in

this subject that the business man needs more than any other is the finding of a certain per cent of a number. The attention is therefore confined to this case. The two further applications that are most needed are discount and interest, and each of these applications is briefly considered.

The supplementary work at the end of the volume may be used for review as the needs of the class indicate.

**Review.** Teachers are urged to give a thorough review of the primary work before the children leave it. It should not be said by a fifth-grade teacher that children are not prepared to take up the work when they enter the class. The review should be both oral and written, and children should show much facility in solving problems accurately and rapidly before they leave this grade. In particular, rapidity and accuracy in the four fundamental operations of addition, subtraction, multiplication, and division, both with integers and with fractions, should be insisted upon. Do not let it be said, after the children are promoted, that they cannot even add, and that they have to count on their fingers when asked for the product of 7 and 8. The actual number facts to be memorized in arithmetic are not many; they are chiefly the addition and multiplication tables. All the rest of the work depends so absolutely upon these facts that no pupil can be said to have been well taught who leaves this grade without perfect familiarity with them. The successful application of these facts to the four operations comes only as the result of such a large amount of drill as will firmly fix the habits involved; and all number work is of secondary importance as compared with this acquisition of the technique of the operations with numbers.

# CHAPTER V

## THE FIFTH SCHOOL YEAR

### THE COURSE OF STUDY

**The leading mathematical features.** A thorough review of the fundamental operations. Common fractions. Operations with denominate numbers. Percentage begun.

**Review.** See the suggestions for review in the preceding grade. In general a new book is begun in this grade, and, if properly arranged, it offers plenty of material for this review of the fundamental operations, with somewhat larger numbers. Accuracy and rapidity insisted upon, as in the preceding grades.

**Nature of the numbers.** Uses of large numbers explained. Operations confined to the common numbers of business, but large numbers should be introduced when they represent genuine conditions.

**Common fractions.** Review the primary work, resorting to objects only when necessary. Tests of divisibility of numbers for the purpose of reducing fractions. Greatest common divisor and least common multiple by factoring. Large and impractical fractions neglected and the emphasis placed upon the four operations with fractions found in ordinary business and in the trades. Mixed numbers treated with fractions. The practical uses of cancellation. Oral and written work with those aliquot parts that are most used in business.

**Decimal fractions.** Development based upon common fractions and the table of United States money. Reduction

to and from common-fraction forms. The four operations completed.

**Denominate numbers.** Reduction. The four operations. In general the work should be limited to the practical compound numbers of two denominations, although three denominations may occasionally be introduced, particularly in finding the difference in dates. Review all tables, adding circular measure and merely referring to the little used and semiobsolete measures.

**Percentage.** In the second half year percentage should be introduced. Especial attention to the important per cents of business, and to their common-fraction equivalents. Particular emphasis on the most important case, — the finding of a per cent of some number. Base and rate also found, but with little attention to these names.

**Nature of the problems.** Our national resources considered. Problems may involve several operations, and explanations should begin to assume scientific form. Operations may be indicated in step form, with symbols. Estimates of answers made in advance, as checks upon the solutions. Bills of goods. Discounts.

**Final review.** See the Fourth School Year.

**Text-book.** Smith's *Intermediate Arithmetic*, Chapter I, Smith's *Grammar School Arithmetic*, Chapter I, and Smith's *Practical Arithmetic*, Chapter I, are identical, and each covers the work of this grade and is conveniently divided for the two parts of the year. The page references apply to each of the books mentioned.

## The Question of Text-Books

The work in Grades V and VI is fairly well settled in most of the school curricula. This work is covered by Smith's *Intermediate Arithmetic*, and also by Chapters I

and II of Smith's *Grammar School Arithmetic* and Smith's *Practical Arithmetic*. Schools wishing a three-book series should introduce the *Intermediate Arithmetic* at the beginning of Grade V. Those wishing a moderate but sufficient course with two books, thus allowing time for some algebra, should introduce the *Grammar School Arithmetic*, using the *Algebra for Beginners* in the latter part of the course. Schools wishing a single book beyond the primary grades, giving an extended treatment of arithmetic, allowing but little time for algebra, and providing for abundant exercises, should introduce Smith's *Practical Arithmetic*, which supplies material for the last four grades of the elementary school.

## Details of the First Half of the Fifth Year's Work

**Notation and numeration** (Page 2). In beginning a new book it is well to review the fundamental operations. The greatest defect shown by children of the present day in arithmetic is their inability to perform the elementary operations with rapidity and accuracy. Therefore the time is well spent that is given to frequent reviews of this work. It is also recommended that two or three weeks at the beginning of each school year be devoted to a rapid and systematic review of the necessary tables learned in the primary grades. The children now come to meet large numbers in their work in geography, and in reading the newspapers. It is seldom necessary, however, to know the names above billions. While the examples in general require moderate sized numbers, it is advisable at this time to show the class some of the uses for large numbers; hence the large numbers on the first few pages. It is

hoped that teachers will endeavor to eliminate the vulgarism of "aught" instead of "naught" (see page 24 of this handbook).

**Addition** (PAGE 6). Teachers are urged to follow the suggestion of § 12, leading the children to read a column like a word. It is also essential to accuracy that children should always check their work. If they form this habit, the common complaint of inaccuracy will be removed.

**Subtraction** (PAGE 10). No definite statement is here given as to the method of subtraction. There are at least three or four methods in common use throughout the country. For a book to insist upon one of these would be to confuse the children in a school where another method is working satisfactorily. Teachers should refer to pages 14 and 25 of this handbook for a discussion of the question. It is essential here, as in the other processes, to check the work, thus forming a habit of accuracy.

**Multiplication** (PAGE 14). Teachers in Grade V should read page 37 of this handbook. This process has already been explained in the primary years. It is well to give the complete operations as suggested on page 16, for the purpose of explaining the various steps. Children should not, however, be expected to give any elaborate explanation themselves. The great thing for them is to be able to multiply rapidly and accurately. The process must become mechanical, and they must check their work rapidly so as to be sure of the results. One of the best checks on multiplication is the casting out of nines. This is usually omitted from arithmetics to-day, because many teachers do not care to use it. For those who have good classes it is advisable to give this check as an interesting feature of arithmetic. For teachers who do not know what it is, the following explanation is inserted.

The practical method of determining whether or not a number is divisible by 9 is as follows: Add the numbers represented by the digits and reject (cast out) each 9 as it is reached; the resulting number represents the remainder, or the *excess*, as it is called.

Thus, to determine whether 124,763 is divisible by 9, begin with the units and add to the left, saying: "3, 9 (reject it), 7, 11 (reject 9), 2, 4, 5"; therefore the remainder is 5. Since the order of adding is immaterial, the eye usually groups the nines at once and the remainder is easily detected.

**Check on addition by casting out nines.** Since numbers are always multiples of 9 plus some remainder, they are of the type $9m + r$. By adding numbers of this type the sum is a multiple of 9 plus the sum of the excesses. Therefore *the excess of nines in a sum is equal to the excess in the sum of excesses.*

$$\begin{array}{l} 9m + r \\ 9m' + r' \\ 9m'' + r'' \\ \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \hline 9(m + m' + \cdots) \\ \quad + r + r' + \cdots \end{array}$$

This check is too long to be of any value in addition alone; it is, however, a necessary part of the check on division.

**Check on multiplication by casting out nines.** Any two numbers may be represented by $9m + r$ and $9m' + r'$. Their product is then represented by $9^2 mm' + 9(mr' + m'r) + rr'$, that is, by a multiple of 9 plus $rr'$. Since the excess of nines in this product is the excess in $rr'$, *the excess of nines in any product equals the excess in the product of the excesses.*

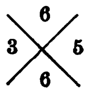
This is better seen by the following example:

$$\begin{array}{r} 38 = \phantom{21}4 \times 9 + 2 \\ 51 = \phantom{21}5 \times 9 + 6 \\ \hline 1938 = 215 \times 9 + 3 \end{array}$$

3 is the excess in $2 \times 6$, that is, the excess in the product of the two excesses, 2 and 6.

This will probably appear too scientific at the first reading. If, however, the mere process is given, without any elaborate explanation, it is easily applied, as is shown in the following example:

$$\begin{array}{r} 437 \\ \underline{129} \\ 3933 \\ 874\phantom{0} \\ \underline{437\phantom{00}} \\ 56373 \end{array}$$




Here the 3 is the excess in 129, the 5 in 437, the upper 6 in the product of 3 and 5, and the lower 6 in 56373. The upper and lower numbers must agree.

The suggestion of multiplication by aliquot parts on page 18 is one that should not be neglected. It is absurd for a child to multiply 25 by 16, using the long process.

**Division** (PAGE 20). The children are now old enough to comprehend the two kinds of division given in § 40. They have probably met them in their primary work, but have very likely forgotten the matter. This is the time to have it well understood. The suggestion concerning the operation of multiplication also applies in division. The full form on page 22 of the arithmetic should be given at the board, but children should at once come to use the simpler form, and the operation should become a mechanical one. In the written work children should be required to obtain the answers in a limited time, rapidity usually meaning accuracy. The check by casting out nines is evidently the reverse of that in multiplication. A definite statement of this check is as follows:

Since the dividend equals the product of the quotient and divisor, plus the remainder, *the excess of nines in the dividend equals the excess in the sum of the excess in the*

*product of the excesses of divisor and quotient, and the excess in the remainder.*

E.g. 561,310,123 ÷ 7654 = 73,335, with a remainder of 4033.

∴ 561,310,123 = 7654 × 73,335 + 4033.

Therefore the excess in 561,310,123, which is 4, equals the excess in 4 × 3, which is 3, plus the excess in 4033, which is 1.

Of course these checks fail to discover any error not affecting the excess of nines, such as an interchange of digits or the addition of 9, but such errors are rare.

**Oral drill** (PAGE 24). At least five minutes of every recitation period should be given to oral work of the kind suggested on pages 24 and 25. The best oral work is, of course, that which the teachers give without reference to any book. When this is given quickly and with interest, the class soon comes to enjoy rapid work.

**The applied examples.** These now come to refer to some of the great industries of the country. It is a poor commentary on modern teaching if we cannot get just as good mental discipline out of modern problems that tell the story of our country as out of problems that are entirely obsolete, and that tell of business conditions of centuries ago. Boys and girls in the city need to know something of the country life, and hence these problems on our country's great resources and industries. Children in the country will be even more interested in knowing what is done in an agricultural way in other parts of America. But few examples involving large numbers are here given, but these are all genuine and show the uses for such numbers. To neglect such work is unwarranted; to give it without practical problems to show its significance is to make it stupid.

**Factors and multiples** (PAGE 30). Formerly a great amount of time was given to subjects like greatest common divisor. This was because the fractions used to be much larger

than at present. The decimal fraction began to be used in the seventeenth century and became common about 1800, and that did away with the necessity for the very large common fractions of the Middle Ages. If the common fractions are not large, the greatest common divisor is not necessary to reduce them to lowest terms, and hence the old treatment of this subject is no longer necessary. It is of much importance to know the tests of divisibility given on pages 32 and 33. When we wish to reduce a fraction to lowest terms, we simply cancel common factors, and hence it is necessary to know what these factors are. For the other treatment of greatest common divisor, see page 92 of this handbook.

In the same way it was formerly necessary to know a great deal about the least common multiple. This was because the fractions were so large that, in order to add them, it was a difficult matter to find the least common denominator. With the simple business fractions of to-day no more elaborate treatment is necessary than that given on page 35.

The tests of divisibility given on page 33 cannot be scientifically explained to children at this time. For the benefit of teachers, however, who should know the reasons involved, the following explanations are inserted.

I. *2 is a factor of a number if it is a factor of the number represented by its last digit, and not otherwise.*

1. Any number has the form $10a + b$, where $b$ has any value from 0 to 9 inclusive, and $a$ has any value from and including 0.

(E.g. $7036 = 10 \times 703 + 6$, and $3 = 10 \times 0 + 3$.)

2. 2 is a factor of 10, and hence of $10a$.

3. $\therefore$ 2 is a factor of $10a + b$ if it is a factor of $b$.

4. Otherwise 2 is not a factor of $10a + b$, for $\frac{10a+b}{2} = 5a + \frac{b}{2}$, and $\frac{b}{2}$ is not an integer.

II. *4 is a factor of a number if it is a factor of the number represented by its last two digits, and not otherwise.*

Any number has the form $100a + 10b + c$, where, etc. The proof is similar to that of I, for $100a$ is always divisible by 4.

III. *8 is a factor of a number if it is a factor of the number represented by its last three digits, and not otherwise.*

The proof is similar to the proofs of I and II.

IV. *5 is a factor of a number if it is a factor of the number represented by its last digit, and not otherwise.*

The proof is similar to that of I, for $10a + b$ is made up of two parts, and $10a$ is always divisible by 5.

V. *9 is a factor of a number if it is a factor of the sum of the numbers represented by its digits, and not otherwise.*

1. Any number may be represented by
$$a + 10b + 100c + 1000d + 10000e + \cdots,$$
where $a$ represents the units' digit, $b$ the tens', $\cdots$.
(E.g. in 7024, $a = 4$, $b = 2$, $c = 0$, $d = 7$, $e = 0$, $\cdots$.)
2. Or by $a + 9b + b + 99c + c + 999d + d + 9999e + e + \cdots$.
3. Or by $9b + 99c + 999d + \cdots + a + b + c + d + \cdots$.
4. Or by a multiple of 9, plus the sum of the numbers represented by the digits.
5. $\therefore$ 9 is a factor if it is a factor of the latter, and not otherwise.

VI. *3 is a factor of a number if it is a factor of the sum of the numbers represented by its digits, and not otherwise.*

In the proof, the first three steps are the same as the first three steps of V. The rest is easy.

VII. *6 is a factor of a number if 2 is a factor of the number represented by the last digit, and if 3 is a factor of the sum of the numbers represented by its digits, and not otherwise.*

For, since $6 = 2 \times 3$, if the number is divisible by 2 and 3, it is divisible by 6.

VIII. *11 is a factor of a number if it is a factor of the difference between the sums of the numbers represented by the odd and the even orders of digits, and not otherwise.*

E.g. 14619 is divisible by 11, since $(1 + 6 + 9) - (4 + 1)$ is divisible by 11.

*Proof.* 1. Any number may be represented by

$$a + 10b + 100c + 1000d + 10000e + \cdots.$$

2. Or by $a + 11b - b + 99c + c + 1001d - d + 9999e + e + \cdots.$

3. Or by $11b + 99c + 1001d + 9999e + \cdots$

$$+ (a + c + e + \cdots) - (b + d + \cdots).$$

4. Or by a multiple of 11, plus the difference between the sums of the numbers represented by the odd and the even orders of digits.

5. $\therefore$ 11 is a factor if it is a factor of the latter, and not otherwise.

IX. *There is no simple method of testing divisibility by 7.*

Most of the above explanation is from Beman and Smith's *Higher Arithmetic* (Ginn & Company), a very helpful book for grade teachers.

**Common fractions** (PAGE 36). The effort is made in this book to emphasize the fractions in ordinary business. It would not, however, be advisable to limit the work entirely to that field, and hence certain other fractions are given. Nevertheless it is generally agreed by the best teachers that it is much better to drill thoroughly upon the fractions that children need than to put in a great amount of time upon absurdly large fractions.

It will probably be necessary to use some objective work in dealing with fractions. Such work is suggested on pages 36–66. No single class of objects should be used to the exclusion of others equally good. It is a serious mistake to confine the work entirely to the ratio method, or to any one set of blocks, diagrams, or charts. Children should see that they may have fractional parts of any object, and that one object may be half as large as another, without

reference to any particular class of things. Sticks like those pictured on page 39 are easily made and are valuable for the purpose there given, but should not be used exclusively.

**The least common denominator.** This is sufficiently treated on page 44. As before said, the subject is not so important for large fractions as it used to be. In the business world it rarely happens that two fractions have to be added whose least common denominator cannot be detected at sight.

**Cancellation.** As suggested on pages 54–55, this should be resorted to whenever possible. The general theory of the subject is more fully given on page 58. In order that children may see the great advantage of the subject, examples of a practical nature are given on pages 60–61.

**Operations with fractions.** It is a frequent subject of complaint on the part of teachers that children who have once learned the reason involved in dividing one common fraction by another very soon forget it. There should really be no complaint whatever with respect to this fact, for such a piece of theory is not expected to be remembered. The work on pages 62–64 is such as to bring out very clearly the reason involved. A child should see that he does not accept the rule of § 122 without a clear understanding of the reason involved. Once this reason is understood, however, the process should become mechanical, and a teacher should not expect a child to give the mathematical principles involved. It would be very difficult for a good computer in a bank to tell the reason involved in half of his computations, — nor is it at all necessary that he should do so.

The advantage of multiplying by aliquot parts, as set forth on page 67, should be manifest to all. The clerk in

a store that needs pencil and paper to solve examples like those on page 67 would not hold his position a week.

The compound and complex fractions given on page 70 are not of very great value. Most school courses require the subject, but a brief treatment is all that is necessary.

It is extremely necessary that children should handle the common business fractions as business people handle them. Therefore such exercises as those on pages 73–75 have been prepared to allow this treatment.

**Decimal fractions** (PAGE 76). The children have already met this subject in the primary grades. They have handled numbers written as dollars and cents in the early part of this grade. There is, therefore, not much that is new in the earlier treatment. It is suggested that very long decimal fractions be omitted. The essential thing is that decimals to three places be thoroughly understood. It will be seen, as usual in this series of books, that the oral exercises at the top of the page give sufficient drill to bring out the theory in the middle of the page. These exercises accomplish two purposes, — offering a drill on the preceding work and leading to the development of the theory.

Since the work of this half year is largely given to common fractions, no more elaborate theory of decimals is necessary than is given on pages 76–87. Teachers should question the pupils as to the meaning of the note under § 142, leading them to see that this refers to fractions in their lowest terms, and not to those like $\frac{3}{15}$, $\frac{7}{14}$, and $\frac{5}{20}$.

In the written exercises on page 88 a suggestion is made that the approximate answer should be given before the work is begun. If teachers would only accustom pupils to do this whenever possible, they would avoid some of the very absurd results not infrequently met in written work. Every good computer should know before he begins about

what the answer is to be. If he finds that his result is far out of the way, he should at once infer that he has made some mistake, and should examine his work again.

**Denominate numbers** (PAGE 90). The tables are here given once more for reference. Those that are of importance are printed in heavy type; the others may be given if the teacher thinks it advisable.

It should be remembered that compound numbers are not used as extensively as formerly. It is very seldom that we see them involving more than two denominations. The very long examples formerly seen in the old arithmetics are now entirely obsolete. Every one recognizes that it is better to give but few problems of the tedious type, and more of the kind that the world is now using. We have so much need for drill in the common processes that time saved in the treatment of elaborate compound numbers may well be applied where it is more needed. The table of Troy weight, for example, with problems depending upon it, may well be omitted at present. Before 1875 it was commonly used in scientific laboratories in this country, and there was therefore some reason for teaching it. At present, however, it has been replaced by the metric system, except in the jeweler's trade, and it has no place in the crowded school curriculum. The same may be said of the apothecary's tables, which would only confuse a child to no purpose. If he goes into the drug business he will easily learn the tables when he needs them, for he will have forgotten them if he ever learned them in school.

Formerly much attention was given to the table of time, with elaborate rules for finding leap years and Easter Sunday. At present but little of this is necessary, on account of the cheapness of calendars. If the teacher wishes to give a slightly different rule for leap year, it is that years

whose numbers are divisible by 4 are leap years, except those centennial (hundred-year) numbers which are not divisible by 400. Thus 1900 was not a leap year, 1904 is one, as is every fourth year for two centuries thereafter, 2000 being divisible by 400. But 2100 is not a leap year, for it is a centennial number not divisible by 400. For practical purposes, for the next two centuries, the test of divisibility by 4 is sufficient, the more technical rule belonging to astronomy rather than arithmetic.

## Details of the Second Half of the Fifth Year's Work

**Review of decimals** (Page 105). The class has now completed the general theory of decimal fractions, with the exception of the work with a decimal multiplier and a decimal divisor. This work is introduced on page 106. If the teacher bases the work upon common fractions, as explained in the book, very little difficulty will be experienced. The plan suggested for division, page 110, is much to be preferred over the old one of pointing off as many places in the quotient as the places in the dividend exceed those in the divisor.

**How to solve problems** (Page 115). Since the work now involves the solution of a large number of problems, it is advisable to give a little instruction as to accurate forms in solving, differentiating clearly between the actual computation and the step form of the analysis. The teacher should read § 195 with care. The problems are such as have to do with our national resources. Teachers are urged to introduce local problems to supplement those given in the book, thus making the subject seem even more real to the children.

**Pictures of magnitudes** (PAGE 120). At the present time a great deal of work is given in the newspapers and business manuals in the representation of magnitudes by means of lines and squares. Any table of statistics furnishes matter for such exercises. While it is not well to carry this work to any great extreme, the children should be able to represent number changes in this way, and to understand them when so represented.

**Bills and receipts** (PAGE 125). This subject, already presented in the *Primary Arithmetic*, is here reviewed and extended. In order to make the topic more real it is advisable to have the children do some work with bills for school supplies, and make out imaginary accounts.

**Percentage** (PAGE 129). It is probable that the children have met percentage previously in the primary work. The essential thing at the beginning of the subject is that they should understand that no new principles are involved, but that this is simply a form of work with decimal fractions. They should understand that per cent means merely hundredths (see § 202). They should also be familiar with the interchange of the per cent forms, decimal fractions, and common fractions, as is fully set forth on pages 129–136. In general it is not advisable at this time to introduce formulas. It is better that the pupil analyze each problem as it arises, reserving for the next grade the use of $x$ for the unknown quantity.

**Discount** (PAGE 147). The most important single application of percentage is discount, and this is introduced at the present time, reserving interest for the next half grade (see this handbook, pages 73, 74).

**Review of denominate numbers** (PAGE 149). Such reviews should be undertaken frequently, so as to keep the important tables in mind. On the other hand, unimportant

tables which are rarely used should not be made the basis of very much drill. These tables are changing more or less all the time. For example, the one concerning paper (§ 225) has changed so that 500 sheets are almost always called a ream at the present time.

**Measurements and comparisons** (PAGE 151). The names of the common plane figures have been used by the children since they entered school. It is now well that they should meet the definitions, not so much for the purpose of learning them as of realizing what an exact definition means. The rule for finding areas of triangles and parallelograms is best developed by the use of paper cutting (page 154). The approximate measures given on page 157 need not be learned unless local conditions make it advisable.

**Review.** Before the children leave this grade teachers should insist upon a systematic and rapid review. They should know before the pupils are passed to the sixth grade that they are accurate and reasonably rapid, and that they readily handle the fundamental operations with the various kinds of numbers.

# CHAPTER VI

## THE SIXTH SCHOOL YEAR

### The Course of Study

**The leading mathematical features.** Percentage and its applications, particularly to discount, profit and loss, commission, and interest. Ratio and simple proportion.

**Review.** This should be undertaken in connection with a study of the formal solution of problems. Review the tables and the fundamental operations.

**General solution of problems.** Analysis, unitary analysis, and the equation with one unknown quantity.

**Percentage.** The subject should be continued. The equation may now be used to elucidate principles. Especial attention to the common per cents and fractions of business. Profit and loss. The direct case in commercial or trade discount, including several discounts, and the application of the subject to bills of goods. The direct case of commission. In the first half year, interest involving years or half years. In the second half year, the time should be expressed in years, months, and days. Bank discount.

**Checks, bills, notes, and receipts.** An introductory treatment in the first half year, with an extension to include promissory notes in the second half year.

**Denominate numbers reviewed.** See the preceding grades.

**Ratio and simple proportion.** Particular attention to the underlying principles and to the genuine applications to the life of the people of America to-day.

**Nature of the problems.** The child's interests are now becoming so broad as to allow a wider range of genuine applications to life. Mere puzzles and problems that give a false idea of business should give place to the problems of real life. When large numbers are used they should generally refer to real concrete cases.

**Text-book.** Smith's *Intermediate Arithmetic*, Chapter II, Smith's *Grammar School Arithmetic*, Chapter II, and Smith's *Practical Arithmetic*, Chapter II, all cover the work of this grade, and are conveniently divided for the two parts of the year. The page references apply to each of the books mentioned.

## Details of the First Half of the Sixth Year's Work

**The general solution of problems** (Page 159). As an introduction to percentage and its applications, the pupil should now undertake a systematic study of the methods of solving problems. This enables the teacher to give a review of the preceding work, and also to prepare the pupil for that which immediately follows. Careful attention should be given to the suggestions on pages 159, 160, 164, 168, 172, 174, 178, and 179. It should be remembered that there is really but one method for solving all problems, and that is by the use of common sense. If a child cares to solve by some form different from that suggested in the book, he should be allowed to do so. In case his solution is more brief and clear, he should be commended; otherwise he should be encouraged to see the advantage of the special form suggested. In any event, his written work should be accurate, and if it is put in step form, for the purposes of

explanation, the dollar signs and other symbols should be in their proper places.

**The unitary analysis.** This subject, as suggested on page 168, is valuable. The work suggested on page 169 is better than the plan of solution by compound proportion, but should not be made the subject of very much drill, for the reason that the practical problems under this subject are relatively few.

**The equation method.** This is introduced on page 176. The question of the introduction of algebra into the grammar schools has long been debated. Teachers have generally come to feel that the only object in putting algebra into arithmetic is to make the children familiar with representing the unknown quantity by the letter $x$. It is too early to introduce algebra in this grade, and the subject is not at all necessary at this time. It is better to postpone it until the latter part of the course than to take the attention from the calculations of arithmetic. Therefore teachers are advised to introduce just so much of the equation as is presented on pages 178–180, and to apply this subject as set forth in the examples in percentage upon the following pages.

**Percentage** (PAGE 181). This subject has already been introduced in the first half of this grade, and it is now taken up in a more thorough way. The introduction of the letter $x$ makes the theory a little clearer, although teachers who prefer to use the ordinary analysis, without the $x$, may easily do so without interfering with the work. Percentage and its applications constitute the leading feature of this half year's work.

**Discount** (PAGE 191). Special attention is still given to this subject, as being the first application of percentage which a child is liable to meet (see this handbook, page 67).

**Profit and loss on purchases** (PAGE 194). The child is now old enough to understand the application of percentage from the standpoint of the merchant, and this is the second form of application which he is liable to meet. It is important, however, that pupils should feel that no new mathematical principle is involved in topics of this kind. The old idea of burdening the child with new rules for every new topic is happily obsolete. For example, the illustrative problem on page 198 might be solved by rule, but it is in every way more desirable to take one of the solutions there given, which shows the reason for the process and which makes the child independent of all rules. That the matter may be more fully appreciated by teachers, this problem is here repeated and the discussion is extended.

How much above cost must a man mark goods in order to take off 25% from the marked price and still make a profit of 20%?

He sells for marked price − 25% of it, or 75% of it.
He also sells for cost + 20% of cost, or 1.20 (120%) of cost.
Therefore 75% of marked price = 1.20 of cost,
and therefore " " = 1.20 of cost ÷ 75% = 1.60 of cost.
That is, the marked price is 160% of cost, or 60% above cost.

Some prefer to solve the problem like this:

Let $1 = the cost.
Then $1.20 = selling price, which is 25% below the marked price.
Therefore 75% of marked price = $1.20,
and 1% " " " = $1.20 ÷ 75 = $0.16,
and 100% " " " = 100 × $0.16 = $1.60,
and $1.60 is 60% more than $1.

The only objection to this second method is that children are liable to confuse *cents* and *per cent*. The advantage of the solution is that the quantities seem more concrete and

tangible than in the first solution. The problem may also be solved algebraically, thus:

Let $m =$ the marked price,
and $c =$ the cost.
Then $m - 25\%\ m = 75\%\ m$, the selling price;
and $c + 20\%\ c = 120\%\ c$, also the selling price.
Hence $75\%\ m = 120\%\ c$,
and $m = 120\%\ c \div 75\% = 160\%\ c$.
Hence the marked price is 160% of the cost, or 60% above cost.

It is, however, a good general principle that the teacher should select the one method that seems best adapted to the class in question, and should emphasize that one. To use several methods is usually confusing.

**Commission** (PAGE 199). This is also an application from the standpoint of the merchant or of the farmer who sends produce to the city. The principle involved is, however, one with which the pupil is already entirely familiar.

**Interest** (PAGE 202). This subject is not met so frequently as the three preceding ones, since we find 10% discount or profit or commission more often than we reckon interest. It is therefore merely introduced here, and is made the subject of more extended treatment in the next half year.

**Checks and receipts** (PAGE 204). This subject should be made real by actually writing out checks and receipts on paper, or by filling in blanks which the school should purchase for this purpose. The subject of banks is taken up later in the course. Many pupils leave school in the sixth grade, and therefore need to understand these two forms at this time.

**Denominate numbers reviewed** (PAGE 206). The necessary tables have now been learned, but they should be made the subject of frequent review throughout the course.

## Details of the Second Half of the Sixth Year's Work

**Simple interest** (Page 211). The pupils have already met the subject of simple interest in the first half of this year's work. It is now taken up again and somewhat extended, being completed later in the course at a time when children appreciate its technicalities more fully. Much practice is required in the writing of promissory notes, so that these business forms may become familiar. The rate of interest varies so much in different parts of the country that it will be necessary for teachers to supplement the examples given in any book. The rate in many parts of the country is limited to six per cent, and it would give a wrong idea of business in those localities if a higher rate were given. Days of grace are now obsolete in most parts of the country, but where they are still given children should be instructed concerning them.

**Ratio** (Page 217). The subject of ratio has usually been introduced in text-books merely as leading to proportion. An effort is here made to show that the subject has some use by itself. Sometimes teachers attempt to base the whole theory of fractions upon ratio. This is, however, a mistake psychologically, although logically it is all right. The adult whose mind allows him to analyze a question carefully will recognize that all numbers are ratios, $\frac{3}{4}$ being the ratio of 3 to 4, and 7 being the ratio of 7 to 1. This, however, is too scientific for a child to grasp, and hence it is better to keep the subject of ratio by itself, as is customary with most writers of arithmetic.

**Proportion** (Page 223). The subject of proportion is not as important as it formerly was. We have now other and better methods of solving examples, as explained

on pages 159–180. A proportion is merely an equation anyway, and the simple equation is easier than the proportion form. In the Middle Ages, when teachers were looking for a universal rule by which to solve problems, they made much of the "Rule of Three." Somewhat later it was found that the Rule of Three was merely the old Greek proportion. The chapter on the Rule of Three therefore dropped out of arithmetic, being replaced by the one on proportion. There are not many cases in which this subject is necessary, the chief ones being those involving similar figures, as on page 224. It will be noticed, however, that the problems given under this subject in this book are more genuine than those usually found in arithmetics. One of the most important questions that arises in teaching the subject is that of finding the height of buildings and trees by means of shadows. This is suggested on page 225, and forms an interesting practical problem in connection with finding the dimensions of the school building.

**Measures** (PAGE 228). The tables of measures are now substantially completed, and should be reviewed at this time. The work should be made as practical as possible, as is done in this part of the arithmetic. On pages 229 and 230 the children are led to compare the weights of certain substances with that of water. The teacher may feel that it is wise to carry this subject further and to speak of specific gravity, a subject not at all difficult.

The specific gravity of any substance is merely the ratio of the weight of that substance to the weight of an equal volume of some other substance taken as a standard. In the case of solids and liquids distilled water is usually taken as the standard. Thus the specific gravity of mercury, of which 1 cu. ft. weighs 13596 oz., is 13.596,

because this is the ratio of the weight of a cubic foot of the substance to the weight of a cubic foot of water; that is,

$$13596 \text{ oz.} : 1000 \text{ oz.} = 13.596.$$

In the case of gases either hydrogen or air is usually taken as the standard. The subject is of great importance in finding whether a given substance is or is not pure. The specific gravities of a few substances may be helpful to the teacher in dictating extra problems, — a feature that often adds interest to the work.

SPECIFIC GRAVITIES, REFERRED TO WATER

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Copper | 8.9 | Nickel | 8.9 | Cork | 0.24 | Alcohol | 0.79 |
| Gold | 19.3 | Silver | 10.5 | Granite | 2.7 | Petroleum | 0.7 |
| Lead | 11.3 | Sulphur | 2.0 | Steel | 7.8 | Mercury | 13.596 |

SPECIFIC GRAVITIES, REFERRED TO HYDROGEN

| | | | | | |
|---|---|---|---|---|---|
| Air | 14.43 | Oxygen | 15.95 | Coal gas | 6 |

SPECIFIC GRAVITIES, REFERRED TO AIR

| | | | | | |
|---|---|---|---|---|---|
| Oxygen | 1.11 | Hydrogen | 0.07 | Chlorine gas | 2.44 |

In the following pages (from page 232) the children are taken into the forests, are told something of the extent of our timber products, and are shown how the height of a tree is found, and how the lumber is measured after it is cut. They are instructed as to the amount of lumber necessary for a small house, these numbers being taken from the actual computations of a carpenter. They then investigate the plastering of houses, the laying of carpets, the papering, and the laying out of the garden and grounds. It is probable that most teachers will supplement this work by problems concerning the school premises.

The method of measuring the height of trees, given on page 233, is one in actual use to-day by woodsmen, and every teacher will probably have the pupils measure the

height of trees or of the schoolhouse in this way. A string fastened at *C*, with a small weight at the other end so as to make a plumb line, will lie along *CB* when the side *AB* of the triangle is level.

In studying board measure (page 234) it is a good plan to find how many board feet there are in the floor of the schoolroom, and to tell the pupils the local cost of such lumber, letting them compute the value of the floor. Not only is this a good exercise in computation, but it gives a helpful piece of information as to values.

The house illustrated on page 236 is one recommended by a government report for a small farm cottage. The measurements are taken from the builder's plans and specifications. It sometimes happens that a building is being erected near the school when this subject is being studied, and teachers have often found it very helpful to have the class compute the cost. Builders are always glad to give the necessary information to teachers.

The subjects of carpeting and papering (pages 239–240) are always complicated by the question of the pattern, and therefore are not so practical as at first seems to be the case. A few problems are, however, of value as showing the general principles involved.

The problem on page 241, relating to laying out a garden and grounds, is taken from an actual case. The grapevines (Ex. 2) are to be set out as shown by the small crosses, and hence the problem itself is abridged in statement. It is a good plan to have the picture drawn to a scale twice as large as that used in the book, allowing the children to try to improve the design. This brings in some work in proportion and in exact measuring.

**General business applications** (Page 242). At this time the children are interested in the large industries of our

country, and hence a few of these are here studied. It is well to correlate this work with that of geography, where this can reasonably be done. When the children are studying about the coal mines it is advantageous to consider page 243, and so for other subjects. These problems are all based upon actual conditions, most of them having been suggested by men who are in the business specified. The business conditions of the place in which the school is located should occupy the attention of the class during a portion of this half year.

**Review.** Before leaving this grade pupils should be given a thorough review upon the essential features. This should include rapid and accurate work in the fundamental operations with integers and fractions. The frequent complaint that children in the seventh grade do not know the multiplication and addition tables, and cannot rapidly perform the fundamental operations, would largely be removed if teachers would conduct a thorough review at the beginning and end of each school year. This review should not only be oral but should also be written, a time limit being placed upon the work.

It cannot too frequently be recalled to the attention of teachers that the most severe criticism that falls upon them comes not from the principal nor from the parents, but from the teacher of the next higher grade. This includes the teacher in the last year of the high school who is criticised by the college teacher, the professor in the senior year of college who is criticised by the professor in charge of the graduate courses, and the latter by the professional school, and so on. Hence every teacher should see to it that no pupil leaves the grade who is not at least fairly prepared, and that no one begins the year without being refreshed upon the preceding work.

# CHAPTER VII

## THE SEVENTH SCHOOL YEAR

### Differentiation in Courses

There is less uniformity in courses of study in the seventh and eighth school years in various parts of the country than in the preceding grades. In some schools less time is given to mathematics in the seventh and eighth grades; in others, part of the time is allotted to algebra, or to elementary geometry, or to both; in others, arithmetic has the same time and attention as in the earlier years. Smith's arithmetics meet all of these conditions. Chapter III of the *Grammar School Arithmetic* offers ample work for schools that somewhat diminish the time allowed to arithmetic, the *Algebra for Beginners* supplementing this course and covering the work necessary for a year's study of this subject. The *Advanced Arithmetic*, or Chapters III and IV of the *Practical Arithmetic*, meets the needs of schools that devote two full years to completing the subject. The following outline is adapted to the last-named plan. Schools desiring to follow the other plan will find the books above mentioned arranged to meet their needs, whether they take arithmetic in the seventh or eighth grade, or divide the time in both grades between arithmetic and algebra. The variation in the attainments of children in different parts of the country, or even in the same city, is so great that no general rule can be laid down for all schools.

## The Course of Study

**The leading mathematical features.** A general view of arithmetic, with particular attention to the foundation principles and the practical short methods. Ratio and proportion completed. Broader view of the applications.

**Writing numbers.** Particular attention to the various types of numbers, and especially the relation between integers, fractions, and compound numbers. Uniform and varying scales.

**The fundamental operations.** All of these reviewed with integers, common and decimal fractions, and compound numbers. Particular attention to rapidity and accuracy. Checks upon all operations.

**The practical short methods.** Rapid oral work. Aliquot parts. The genuine cases needed in the shop and store.

**The most important measures reviewed.** Application to practical problems.

**Longitude and time.** Make the subject as practical as possible. Recognize the general use of standard time in most parts of the country.

**Ratio and proportion.** Applications to real business problems of this country. Schools wishing to introduce compound proportion should do so at this time. The reasons involved should be clearly understood.

**Nature of the problems.** These should represent actual business conditions of the present time. There should also be enough abstract drill work to secure rapidity of operation.

**Text-book.** Smith's *Advanced Arithmetic*, Chapter I, or *Practical Arithmetic*, Chapter III, exactly covers this work. For schools wishing a sufficient amount of work, but not allowing enough time to accomplish all that is here laid down, Smith's *Grammar School Arithmetic*, Chapter III,

first half, furnishes enough material. The page references are now to the *Advanced Arithmetic*, *Practical Arithmetic*, and *Grammar School Arithmetic*, briefly designated by *Adv.*, *Prac.*, *G. S.*

### A Course for completing Arithmetic in the Seventh Grade

**The leading mathematical features.** A general review of arithmetic and a study of the most important business applications. This may be made in the seventh grade, or it may be combined with algebra and extended over the seventh and eighth grades, or it may be extended by itself over the seventh and eighth grades.

**Writing numbers.** Special reference to the relation between numbers on different scales.

**The operations reviewed.** The work should show the relation between the operations with integers, fractions, and compound numbers, and the general underlying principles involved.

**The practical short methods.** Aliquot parts. The common fractions of business.

**Longitude and time.** Brief treatment, with discussion of standard time.

**Measures.** Review all tables and apply them to practical problems.

**Percentage.** Review the essential features, including interest.

**Business operations.** Take the pupil into business. Bank accounts, partial payments, discounts, partnership, insurance, taxes, and corporations.

**Business and household accounts.** Bills and receipts. Accounts balanced.

**Business papers.** Checks and stubs, promissory notes.

**Exchange.** Make the subject practical, using the common forms. Money orders, bank and commercial drafts, checks.

**Banking.** Opening accounts. Various forms of banks. Compound interest in savings banks.

**Corporations.** Organization and management. Stocks and bonds.

**Taxes.** Customs, internal revenue. State and local taxes. Government expenses.

**Mensuration.** Circle, sphere, prism, cylinder, pyramid, cone. Square root.

**The metric system.** Practical measurements showing the value of the system.

**Text-book.** Smith's *Grammar School Arithmetic*, Chapter III, exactly covers this course.

### Details of the First Half of the Seventh Year's Work

**General features of the work.** A general review of arithmetic. The great criticism made upon pupils as they leave the grammar school is that they are deficient in the fundamental operations. This important work is, therefore, again reviewed at the beginning of this school year. The review, however, is on a considerably higher plane, the relations between the various classes of numbers being shown under each operation.

**Writing numbers** (*Adv.*, Page 1; *Prac.*, Page 265; *G. S.*, Page 253). It is a good plan to interest the pupils in the historical features of this work. Our common numerals were first found in very crude form in a cave inscription near Bombay, India. They were cut in the rock about 250 B.C. do not know when the zero was added to the system,

but it definitely appeared in the eighth century. These numerals were carried to Bagdad just before 800 A.D. That was then the great center of Arab learning, and from there the numerals spread westward to Europe. They were first prominently introduced into Italy in 1202, when Leonardo of Pisa, a young man who had been educated in a Moorish school in North Africa, wrote his great mathematical book. They did not become universal, however, until books began to be printed, in the latter part of the fifteenth century.

The first arithmetic was printed in 1478, at Treviso, Italy, and after that the Hindoo-Arabic numerals soon became common. The Roman numerals are of very slight importance at the present time. They are used only for numbering pages or paragraphs, and occasionally for writing dates. In the matter of dates teachers are sometimes in doubt as to writing numbers beyond 1900. The Romans themselves usually wrote these numbers in words. In the Middle Ages a number like 1900 was usually written MDCCCC. It was frequently written MDcccc. In Germany, about the year 1500, it was very commonly written MXVIIII$^c$. Probably the method that has been most common is the first one mentioned. Nevertheless it has of late become a custom to write this number MCM, and to write the number 1908, MCMVIII. This last method is convenient, and is so generally understood that it is legitimate to use it even though it is not a common Roman form. If children are instructed in the matter at all, they should be told that the first and last forms above given are both used at the present time. The following interesting forms may be shown to the children, as illustrating the numbers in actual use from the time of printing until about 1600.

| | |
|---|---|
| Mcccccccviiii for 1609; | Mv$^c$, for 1500; |
| Mcccclxxxx, for 1490; | II$^m$V$^c$, for 2500. |

It is desirable that pupils should understand the difference between a uniform scale and a varying scale. Formerly the world generally used varying scales for all work in denominate numbers. Within the last century the tendency has been to use uniform scales, as is seen in the table of United States money and in the metric system.

If classes are rather well advanced, teachers may think it worth while to explain the modern index notation. This is set forth in a work which is very helpful to grade teachers as well as to those in the high school, — Beman and Smith's *Higher Arithmetic* (Ginn & Company). The explanation is as follows:

In modern science numbers are often used which contain several zeros, for the reason that absolute accuracy of measurement is generally impossible. Thus it is said that the distance from the earth to a certain star is 21,000,000-000,000 miles, but the distance even to within a billion miles is quite unknown. Similarly, the length of a wave of sodium light is said to be 0.0005896 of a millimeter, but the seventh decimal place is doubtful and the subsequent ones are unknown. The naming of these numbers is a matter of little importance, and the writing of them in full is usually unnecessary. Scientists often resort to an *index notation*, in which an integer, sometimes followed by a decimal, is multiplied by a power of 10. Thus, 21,000,000,000,000 may be written $2.1 \times 10^{13}$, or $21 \times 10^{12}$. And since $10^{-1}$ means 0.1, and $10^{-2}$ means 0.01, etc., therefore 0.00000274 may be written $2.74 \times 10^{-6}$.

Since the index notation is now so extensively used in science, and since the limit of necessary counting in financial affairs is met in the billion or trillion, no elaborate system of naming numbers is practically used.

**Addition reviewed** (*Adv.*, PAGE 8; *Prac.*, PAGE 272; *G. S.*, PAGE 260). The two essential features to be considered in this connection are accuracy and rapidity of work and the underlying principle of addition. The pupils should see that the same principle is involved, whatever kinds of numbers are added. If the class is deficient in rapid and accurate addition, the work may be supplemented by problems written upon the blackboard. If the pupils are told to check their work, and if the answers of the class are compared, it will not be necessary for the teachers to have the answers in advance.

As an exercise, it is convenient to prepare a table of multiples of some number in this manner: Write any number, as 4197, on paper, and the same number on a small card, $\boxed{4197}$; place the card above the number and add, thus giving $2 \times 4197$; slide the card down and add again, thus giving $3 \times 4197$, and so on to $10 \times 4197$, when the work checks if the result is 41,970.

**Subtraction reviewed** (*Adv.*, PAGE 12; *Prac.*, PAGE 276; *G. S.*, PAGE 263). The suggestions already given in regard to addition apply with equal force to subtraction. As to the method of subtraction, teachers should refer to page 25 of this handbook. Special attention should be given to oral subtraction. When this is done, however, the examples should be written upon the blackboard or be taken from the book, and not simply read to the class. Business men do not have to subtract numbers of three figures by simply hearing them; they always see them, and then subtract, putting the work on paper. Children should, therefore, be trained in the same way, giving the answer as soon as they see the two numbers, or using pencil and paper if the numbers are long. As suggested on page 12 of the *Advanced Arithmetic*, it is usually better in oral work to begin at the left.

**Multiplication reviewed** (*Adv.*, PAGE 15; *Prac.*, PAGE 279; *G. S.*, PAGE 266). The pupil should again see that the fundamental principle involved is the same whatever kinds of numbers are used. If they do not understand the check by casting out nine's, the teacher may feel that it is worth while to introduce it here. The explanation given on page 57 of this handbook will probably be too difficult for the children, but there is no objection to giving the method, even though the underlying reasons are not fully understood. All computers use this check, and it is so easily applied that it seems a pity that it is not better known. In multiplication by decimal fractions, the explanation given in this chapter will be found so clear that pupils cannot fail to understand it without any difficulty, and they will be independent of any dogmatic rule as to "pointing off as many places in the product as there are in the two factors."

**Division reviewed** (*Adv.*, PAGE 20; *Prac.*, PAGE 284; *G. S.*, PAGE 268). Although the pupils have already met the two cases of division, these cases should here be reviewed. The pupils are now old enough to have no difficulty in distinguishing between the two, and abundant opportunity is given in the problems to show that the distinction is clearly understood. The fact that the fundamental principle involved is the same for all kinds of numbers is clearly brought out in the text-book, and teachers should see that the pupils appreciate it.

It is a frequent complaint that pupils do not understand why it is that the divisor is inverted in the case of fractions. It is quite natural that it should be so, because the theory is too difficult to be remembered by young children. In this grade, however, they are quite prepared to understand the reason involved, which is clearly brought out in the oral exercise preceding this topic.

In the dividing of denominate numbers and of mixed numbers it should be seen that the principles involved are the same as with integers.

**General principles of the operations** (*Adv.*, PAGE 27; *Prac.*, PAGE 291; *G. S.*, PAGE 272). While it is not essential that a child should learn the names given to these fundamental principles in algebra, the laws themselves should be recognized. Of course the pupils have always taken these laws for granted, and the only object in introducing them here is to call attention to them. The law of order is known in algebra as the *commutative* law, and the law of grouping is known as the *associative* law. These names are used in algebra, but they need not be given to the children in this grade.

**The practical short methods** (*Adv.*, PAGE 28; *Prac.*, PAGE 292; *G. S.*, PAGE 273). Some of the older arithmetics used to give a number of tricks of computing. These usually applied only to special questions, and were of but little importance. As is stated in Smith's arithmetics, a great deal of the work of computing is now done by machine. The machine here illustrated costs from $200 to $300. A good machine for multiplying and dividing costs from $100 to $200. There are numerous machines that are not so expensive, and that do more or less of this kind of work. Engineers and others, who have a good deal of calculating to do, generally use tables or a simple instrument known as the *slide rule.*

The short method of addition suggested in the text-book is the only one that is of particular value. A computer soon

learns to read a column of figures without pronouncing to himself any of the numbers. We cannot, in school, train the pupils to do much rapid work of this kind. Those who are going into bookkeeping will soon learn rapidity from constant practice. All that can be hoped for from the time at our disposal in school is that the children shall add accurately, shall check every addition, and shall do all this with reasonable rapidity.

In subtraction the only short method that is of value is the one already suggested in this handbook on page 25. Teachers should always insist upon pupils checking the work.

In multiplication, however, there are numerous short methods that are actually used in the stores and workshops. These should become entirely familiar to the pupils, and abundant drill is furnished in the text-book for accomplishing this result. Children should be able to multiply 125 by 24, or to get $16\frac{2}{3}\%$ of 540, or to find the cost of $5\frac{3}{4}$ yards at 16¢ a yard, without thinking of using a pencil and paper. The text-book offers a good deal of drill upon work of this kind, with the numbers commonly used in everyday business. Numbers like $12\frac{1}{2}$, $33\frac{1}{3}$, $6\frac{1}{4}$, and $75\%$ are used so commonly in business that a great deal of drill involving them should be given.

If the teacher cares to supplement these processes, the following may be added as interesting cases.

*To multiply by numbers differing but little from $10^n$.* For example, to multiply by 997 is to multiply by $(10^3 - 3)$; that is, to annex three ciphers and subtract three times the multiplicand. For example,

$$\begin{aligned} 995 \times 1474 &= 1{,}474{,}000 - \tfrac{1}{2} \text{ of } 14{,}740 \\ &= 1{,}474{,}000 - 7370 = 1{,}466{,}630. \end{aligned}$$

*When two factors lie between* 10 and 20 the product is readily found as follows:

$$14 \times 16 = 10\ (14 + 6) + 4 \times 6 = 224.$$

To prove that this process is general:

1. Any number from 10 to 19 may be represented by
$$10 + a,\ 10 + b, \ldots.$$
2. $$(10 + a)(10 + b) = 100 + 10a + 10b + ab$$
$$= 10\ (\overline{10 + a} + b) + ab.$$

*To square numbers ending in 5.* While not as practical as the problems already given, this has some value. The method is illustrated by $65^2$; it is merely necessary to say,

$$6 \times 7 = 42,\ 5 \times 5 = 25,\ \therefore 65^2 = 4225.$$

To prove that this process is general:

1. Any number ending in 5 may be represented by $10a + 5$, where $a$ may equal 0, 1, 2, ..., 9, 10, ....
2. $$(10a + 5)^2 = 100a^2 + 100a + 25$$
$$= 100a\ (a + 1) + 25.$$
3. That is, the result ends in 25, and the number of hundreds is $a\ (a + 1)$.

*Applications of the formulas for* $(a + b)^2$, $(a + b)\ (a - b)$. In a few problems there is an advantage in recalling the identities

$$(a + b)^2 = a^2 + 2\,ab + b^2,$$
$$(a + b)\ (a - b) = a^2 - b^2.$$

For example, $62^2 = 3600 + 240 + 4 \quad = 3844.$

$$23 \times 17 = (20 + 3)\ (20 - 3) = 400 - 9 = 391.$$

Some teachers may have classes sufficiently advanced to consider a common approximation in multiplication, used by computers. This is easily understood from the following explanation given in Beman and Smith's *Higher Arithmetic.*

In multiplication there is no practical advantage in beginning with the lowest order of units of the multiplier; in fact, there is a decided advantage in beginning with the highest order, as is clearly apparent in approximations in multiplication. The arrangement of work would then be as shown in the annexed example. Since $20 \times 0.009 = 0.18$, the position of the decimal point is at once known, and the rest of the process is apparent.

$$\begin{array}{r@{.}l}
437&189\\
26&93\\
\hline
8743&78\\
2623&134\\
393&4701\\
13&11567\\
\hline
11773&49977
\end{array}$$

The need for approximations in multiplication arises from the fact that perfect measurements are rarely possible in science, and that results beyond two or three decimal places are seldom desired in business. Thus, if the radius of a wheel is known only to 0.001 inch, it is not possible to compute the circumference with any greater degree of accuracy; hence labor would be wasted in seeking a product to more than three decimal places. In such cases all unnecessary work should be omitted, as in the annexed example. This represents the multiplication in the solution of the following problem: To find the circumference of a steel shaft of which the diameter is found by measurement to be 10.48 centimeters. Since it was practical to carry the original measurement only to 0.01 centimeter, the result need be sought only to 0.01. In order to be sure that the result is correct to 0.01, the partial products are carried to 0.001; and in multiplying by 0.04, for example, the effect of the fourth decimal place on the third is kept in mind.

$$\begin{array}{r@{.}l}
10&48\\
3&1416\\
\hline
31&44\\
1&048\\
0&419\ .\\
0&010\ .\ .\\
0&006\ .\ .\ .\\
\hline
32&92
\end{array}$$

The short methods of dividing are somewhat less important than those of multiplying. Nevertheless a considerable amount of oral drill is necessary in this line of work, and this is provided for in the text-book.

With a very strong class, or with a particularly bright pupil, a teacher may be disposed to give a common method of approximate division used by many computers. For example, if the circumference of a shaft is found by measurement to be 32.92 centimeters and it is required to know the diameter, it would be a waste of time to attempt to find the diameter beyond 0.01. Since 10's divided by 10,000's $<$ 0.01's, the last two figures of the dividend will not affect the quotient within two decimal places, and hence may be neglected. Hence, also, the divisor may be considered as 3142 and may be continually contracted. The process is apparent by first examining the complete form in the example below. The advanced student should note how much better for practical purposes the last form is than the others.

$$\begin{array}{rl}
 & \phantom{)}10.48 \\
31416) & 329200 \\
 & \underline{3142} \quad = \text{approximately } 10 \times 3141(6) \\
 & 150 \\
 & \underline{126} \quad = \quad \text{“} \quad 0.4 \text{ of } 314(16) \\
 & 24 \\
 & \underline{24} \quad = \quad \text{“} \quad 0.08 \text{ of } 31(416)
\end{array}$$

This may be further abridged by omitting the partial products, thus:

$$\begin{array}{rl}
 & \phantom{)}10.48 \\
31416) & 329200 \\
 & 150 \\
 & 24
\end{array}$$

**The long-division form of greatest common divisor.** This topic was omitted from the work of the earlier grades, when the greatest common divisor was under consideration, because it is obsolete for practical purposes. If it is to be introduced at all it should have place at this time, when the principles of arithmetic are under consideration. The long-division form was of value before decimal fractions were invented, and when it was necessary to reduce fractions like $\frac{1043}{1341}$ to their lowest terms before operating with them. At the present time we do not meet such fractions in ordinary life, and hence we do not need to find the greatest common divisor of numbers like 1043 and 1341. So unnecessary is this work that the largest college entrance examination board in this country has recently decided not to set any questions upon it even in algebra. Nevertheless there are localities where it seems best to teach the subject still, and hence it is inserted in this handbook, although it is now omitted from the best modern arithmetics. Teachers thus have the material at hand if they need it.

The work depends upon the following principles:

1. *A factor of a number is a factor of any of its multiples.*

That is, a factor of 15 is a factor of $3 \times 15$, $4 \times 15$, and so on. This is evident, because if a number is contained in 15 without a remainder, it will be contained in $7 \times 15$, the quotient being seven times as large.

2. *A factor of each of two numbers is a factor of their sum or difference.*

That is, a factor of 147 and 189 is a factor of their sum or difference. This is evident, as will be seen from the annexed work. If we add we shall have $3 \times (63 + 49)$, and if we subtract we shall have $3 \times (63 - 49)$, the addition and subtraction not disturbing the factor 3.

$$\begin{aligned} 189 &= 3 \times 63 \\ \underline{147} &= \underline{3 \times 49} \end{aligned}$$

The process may be understood by finding the greatest common divisor of two numbers not readily factored, as 1043 and 1341.

1. The g.c.d. cannot be greater than 1043, and is 1043 if that is exactly contained in 1341. But it is not, and therefore 1043 is not the g.c.d.

```
1043)1341(1
     1043
      298)1043(3
           894
           149)298(2
               298
```

2. Because the g.c.d. is a factor of 1043 and of 1341, it is a factor of their difference, 298, by Principle 2.

3. Therefore the g.c.d. cannot be greater than 298, and it is 298 if that is exactly contained in 1043 and 1341. But it is not exactly contained in 1043, and therefore 298 is not the g.c.d.

4. Because the g.c.d. is a factor of 298, it is a factor of $3 \times 298$, or 894 (Principle 1); and because it is a factor of 1043 and of 894 it is a factor of their difference, 149 (Principle 2).

5. Therefore the g.c.d. cannot be greater than 149, and it is 149 if that is exactly contained in 1043 and 1341.

6. Now 149 is exactly contained in 298, as we see by dividing. Hence it is contained in $3 \times 298$, or 894 (Principle 1), and hence in $149 + 894$, or 1043 (Principle 2).

7. And because it has been shown that 149 is a factor of 298 and of 1043, it follows that it is a factor of $298 + 1043$, or 1341 (Principle 2). Hence 149 is the g.c.d.

The following examples may be given if it is thought best to introduce the subject.

*Find the g.c.d. of:*

1. 1535 and 2149. (*Ans.* 307.)
2. 1441 and 1703. ( " 131.)
3. 1688 and 3587. ( " 211.)
4. 1287 and 1573. ( " 143.)
5. 1281 and 1647. ( " 183.)
6. 1710 and 7524. ( " 342.)
7. 6055 and 13,321. ( " 1211.)

*Reduce the following fractions to their lowest terms:*

| | | | |
|---|---|---|---|
| 8. $\frac{513}{855}$. | (*Ans.* $\frac{3}{5}$.) | 9. $\frac{1197}{1539}$. | (*Ans.* $\frac{7}{9}$.) |
| 10. $\frac{699}{1631}$. | ( " $\frac{3}{7}$.) | 11. $\frac{1252}{2191}$. | ( " $\frac{4}{7}$.) |
| 12. $\frac{909}{1212}$. | ( " $\frac{3}{4}$.) | 13. $\frac{535}{1177}$. | ( " $\frac{5}{11}$.) |
| 14. $\frac{1618}{5663}$. | ( " $\frac{2}{7}$.) | 15. $\frac{1006}{4527}$. | ( " $\frac{2}{9}$.) |

In the case of three or more numbers, the g.c.d. of the first two is found, then of that and the next, and so on. Thus the g.c.d. of 363, 605, and 1089 is found as here shown. The g.c.d. of 363 and 605 is 121. The g.c.d. of 121 and 1089 is 121. Hence this is the g.c.d. of all three.

```
363)605(1
    363
    242)363(1
        242
        121)242(2
            242

121)1089(9
    1089
```

*Find the g.c.d. of the following numbers.*

| | |
|---|---|
| 16. 426, 852, and 994. | (*Ans.* 142.) |
| 17. 612, 1224, and 1530. | ( " 306.) |
| 18. 1404, 3510, and 7020. | ( " 702.) |
| 19. 1224, 2448, and 4284. | ( " 612.) |

**Measures** (*Adv.*, PAGE 47; *Prac.*, PAGE 311; *G. S.*, PAGE 281). The tables of measure should all be reviewed at this time. Some historical note concerning a few of the units will be interesting to the pupils. The grain was originally the weight of a grain of wheat. A pennyweight was the weight of the old English penny. The foot was derived from the supposed length of the human foot. The inch was the length of the thumb joint, and is called by the name "thumb" (*pouce*) by the French. The yard was taken at one time as the length of the arm of King Henry II, although approximately the same measure had already been used before that monarch's time. The word "furlong"

means a furrow long, and is an old agricultural measure. The word "mile" is from *mille passuum*, meaning a thousand paces. The word "gill" is from the mediæval Latin *gilla*, a drinking glass. The word "quart" is from the Latin *quartus*, meaning fourth. The other words used in the tables may be looked up by the pupils in any good dictionary.

In general it is not advisable to memorize such facts as the number of cubic inches in a bushel. Pupils who wish to use this information can easily find it in an encyclopedia or dictionary. It is better that the children should learn thoroughly the things that are most needed, instead of burdening their minds with such unimportant details. The essentially new features of measure at this time are the surveyor's tables. Although these are not of great importance to the average citizen, it is useful to know the number of square chains in an acre and the number of links and rods in a chain. It is very easy to make a fairly accurate surveyor's chain by taking a stout cord 66 feet long and tying knots to indicate the rods. By means of such a cord the school premises and adjacent pieces of land should be measured. The pupils should thus come to have a good idea of the size of an acre. The work given on the laying out of public lands (*Adv.*, PAGE 65; *Prac.*, PAGE 327; *G. S.*, PAGE 283) may be omitted in those parts of the country where this method is not used. In the older parts of the country it has never been adopted.

## DETAILS OF THE SECOND HALF OF THE SEVENTH YEAR'S WORK

**General features of the work** (*Adv.*, PAGE 67; *Prac.*, PAGE 329; *G. S.*, PAGE 285). The work in the review of denominate numbers is continued in this half grade. The

particular application is to volumes. It should now be insisted upon that pupils understand clearly the method of finding the volume when the dimensions are given. The forms for this work are accurately given in the text-book, and should be insisted upon.

**Longitude and time** (*Adv.*, PAGE 75; *Prac.*, PAGE 337; *G. S.*, PAGE 287). This subject belongs quite as much to geography as to arithmetic, the mathematical features being relatively simple. The subject is taught not merely for the purpose of getting the answer, but as an assistance in accurate reasoning. Therefore it is hoped that the less accurate methods given in many of the arithmetics will not be followed by teachers. The forms given in Smith's arithmetics are accurate and the explanation is complete. The introduction of standard time and its general adoption in a large part of the civilized world within the past few years has put the teaching of the subject upon a somewhat different basis. At present a good deal of the work is oral, on account of the fact that the difference in time is usually expressed in exact hours. The subject of standard time should be made very real to the pupil, and it is a good plan to mark a map of the United States to show the various divisions of standard time, and place it where all the class can see it. It should be remembered that this system was started by the railroads about 1883, and that these roads find it more convenient to change their time at terminal points. This accounts for the irregularity of the lines separating the divisions. Teachers who wish to know the exact points where the changes of time are made, can ascertain this from the official railway guide. The line changes a little from time to time, as the railroads change their terminal points. The question of the international date line is a question which belongs

to geography rather than to arithmetic. It may, however, be introduced in this connection if desired. The subject is fully discussed in Beman and Smith's *Higher Arithmetic* (Ginn & Company), page 83.

**Percentage reviewed** (*Adv.*, Page 84; *Prac.*, Page 346; *G. S.*, Page 294). Following the plan generally adopted by all modern schools, important subjects like percentage are touched upon several times in this course. One important feature at this time is the oral work with the common per cents of business. Pupils should have so much of this work that they do not feel that pencil and paper are necessary for the solving of easy problems involving the simple per cents.

The terms "base," "rate," and "percentage" are not of great value. The word "percentage" itself is often misunderstood by the children because it is the name for the general subject as well as for one of the numbers considered. The word "base" is rarely used in business in this connection. The older arithmetics used the words "amount" and "difference," but this was because they taught the subject merely by rule, or by formulas, and therefore had to have names for the various numbers involved in the rules. It is much better to pay but little attention to these unimportant names, and to concentrate the pupils' minds upon the understanding of the process. It must always be borne in mind that the two important things in percentage are (1) to find some per cent of a given number and (2) to find what per cent one number is of another. These two cases should be greatly emphasized, as is done in Smith's arithmetics.

**Simple interest reviewed** (*Adv.*, Page 99; *Prac.*, Page 361; *G. S.*, Page 299). This subject has been studied by the pupils in the preceding grades, and is therefore taken up as a review topic here. It is elaborated by the introduction

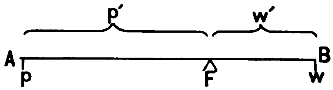
of the six per cent method, the table of time, and the interest table. The six per cent method is valuable when one does not have access to interest tables. In general, however, people who have a great deal of interest to compute, whether they be bankers or private money loaners, make use of tables such as are illustrated in these arithmetics (*Adv.*, PAGE 107; *Prac.*, PAGE 369; *G. S.*, PAGE 302).

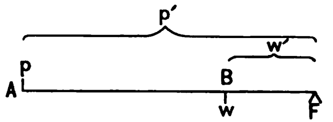
Of course the strictly legal interest is exact interest (*Adv.*, PAGE 108; *Prac.*, PAGE 370; *G. S.*, PAGE 303). This, however, is not commonly used, because it is more difficult to compute without the table, and because it gives a slightly less amount of interest. The difference in interest on small sums is so slight that people are generally in the habit of computing on the basis of 360 days to the year. If a school has a reference library, it is a good plan to put an interest table upon the shelf where pupils may use it. Even if they use it in solving the book problems, the familiarity with the tables is quite as important as familiarity with the ordinary computation on paper.

**Ratio and proportion** (*Adv.*, PAGE 117; *Prac.*, PAGE 379; *G. S.*, PAGE 217). Teachers should review page 74 of this handbook. Formerly problems in proportion were solved merely by rule. This rule was that the product of the means divided by the given extreme equals the required extreme. It is better, however, to use the subject of proportion only for solving practical examples which are not mere puzzles, and to have the pupils solve these problems without reference to such a rule. The old method known as "Cause and Effect" has now gone out of use, because it was really applicable only to problems that were made up for the purpose. In genuine business it was never used. Problems that are really valuable in practical life require no such arbitrary rule.

There are two laws of physics that a teacher may care to introduce in connection with this work, or at least mention as matters of interest (see Beman and Smith's *Higher Arithmetic*, page 94). The first is the *Law of Levers.*

If a bar $AB$ rests on a fulcrum $F$ and has a weight $w$ at $B$, then by exerting enough pressure $p$ at $A$ the weight can be raised. In the first figure the pressure is downward (positive pressure); in the second it is upward (negative pressure).




There is a law in physics that, if $p'$, $w'$ represent the number of units of distance $AF$, $FB$, respectively, and $p$, $w$ the number of units of pressure and weight, respectively, then $\frac{pp'}{ww'} = 1$.

In the first figure $p$, $w$, $p'$, $w'$ are all considered as positive; in the second figure $p$ is considered as negative because the pressure is upward, and $w'$ is considered as negative because it extends the other way from $F$. Hence the ratio $pp' : ww' = 1$ in both cases.

*Example.* Suppose $AF = 25$ in., $FB = 14$ in., in the first figure. What pressure must be applied at $A$ to raise a weight of 30 lbs. at $B$?

1. By the law of levers $\frac{25\,p}{14 \cdot 30} = 1.$

2. Therefore $p = \frac{14 \cdot 30}{25} = 16.8.$

Therefore the pressure must be 16.8 lbs.

The second law is not so well adapted to the pupils of this grade, but it may prove valuable for some. It is *Boyle's Law* for the pressure of gases.

It is proved in physics that if $p$ is the number of units of pressure of a given quantity of gas, and $v$ is the number of units of volume, then $p$ varies inversely as $v$ when the temperature remains constant.

This law was discovered in the seventeenth century by Robert Boyle.

For example, if the volume of a gas is 10 cu. ft. under the ordinary pressure of the atmosphere ("under a pressure of one atmosphere"), it is

$\frac{1}{n}$ as much when the pressure is $n$ times as great,

$n$ times " " " " " " $\frac{1}{n}$ " " "

the temperature always being considered constant.

For example, a toy balloon contains 3 cu. ft. of gas when exposed to a pressure of 1 atmosphere. What is its volume when the pressure is increased to 4 atmospheres? decreased to $\frac{1}{8}$ of an atmosphere?

1. Since the volume varies inversely as the pressure, it is $\frac{1}{4}$ as much when the pressure is 4 times as great.
2. Similarly, it is 8 times as much when the pressure is $\frac{1}{8}$ as great.
3. Therefore the volumes are 0.75 cu. ft. and 24 cu. ft.

The following example may interest some of the boys in the class.

Equal quantities of air are on opposite sides of a piston in a cylinder that is 12 in. long. If the piston moves 3 in. from the center, find the ratios of the pressures. Draw the figure.

**Compound proportion** (*Adv.*, PAGE 124; *Prac.*, PAGE 386). This subject has lost so much of its importance that it is omitted in the *Grammar School Arithmetic*, the problems being solved by unitary analysis (*G. S.*, PAGE 169). Although it has been inserted in the *Advanced Arithmetic* and *Practical Arithmetic*, teachers are advised to give comparatively little attention to it, for the reason that so

few practical problems can be found that cannot better be solved by some other method. The unitary method is somewhat clearer and more direct (see *G. S.*, *Int.*, or *Prac.*, PAGE 169).

**Review problems.** Those given in the *Advanced Arithmetic*, page 128, and the *Practical Arithmetic*, page 390, refer quite largely to the farming industry. More than half of the children in this country belong to rural communities, and the information given in these problems will be very valuable to them. This information is taken from government reports, works on agriculture, and practical testimony of successful farmers. Teachers in rural communities should see that the pupils get from this work something besides the mere computations. In the city schools the information is also valuable, not because the pupils will use it in a practical way, but because they are naturally so ignorant of the great farming industry that information of this kind supplies a lack in their stock of knowledge. There are few industries in the country that have made such progress in a few years as farming. The ideas of scientific fertilizing, of testing milk, of sanitation of the dairy, and of saving what was formerly wasted, all raise very genuine questions to-day in grange meetings and in farmers' journals, and their study from the arithmetical standpoint is far more valuable than many of the inherited problems about farming.

The last three examples in this review (*Adv.*, PAGE 137; *Prac.*, PAGE 399) are given as rather severe tests for the children in closing the year's work. They like to end the work with a few difficult problems, but if these should prove too hard for certain schools, they should of course be omitted. The following are the solutions of Exs. 31 and 33.

**Ex. 31.** If he adds $x$ lb., he will have $(100 + x)$ lb. in all. Since $\frac{95}{200}$ of the 100 lb. was acid phosphate, there were $47\frac{1}{2}$ lb. of this substance at first, and $(47\frac{1}{2} + x)$ lb. after the addition.

Since 50% of the total was acid phosphate, we have

$$50\% \, (100 + x) \text{ lb.} = (47\tfrac{1}{2} + x) \text{ lb.}$$

Hence
$$50 + \tfrac{1}{2}x = 47\tfrac{1}{2} + x,$$
$$2\tfrac{1}{2} = \tfrac{1}{2}x,$$
$$5 = x.$$

Therefore he added 5 lb. of acid phosphate.

**Ex. 33.** Since 25% + 5% + 20% is blood, soda, and potash, the acid phosphate must be 100% − 50%, or 50%, of the mixture.

Since the sulphate of potash is 20% of the mixture, 400 lb. = 20% (or $\frac{1}{5}$) of the mixture, and 2000 lb. = the mixture.

Therefore 1000 lb. = the acid phosphate.

Therefore, if he uses $x$ lb. less, he will use $(1000 - x)$ lb. of acid phosphate, and the mixture will weigh $(2000 - x)$ lb.

But the acid phosphate is to be 45% of the mixture. Therefore

$$45\% \, (2000 - x) = 1000 - x,$$
$$900 - 45\% \, x = 1000 - x,$$
$$55\% \, x = 100,$$
$$x = \tfrac{100}{55} \times 100,$$
$$x = 181\tfrac{9}{11}.$$

Therefore he uses $181\frac{9}{11}$ lb. less acid phosphate.

These three cases are taken from standard works on agriculture, although they are purposely more difficult than the ordinary ones that will be met.

# CHAPTER VIII

## THE EIGHTH SCHOOL YEAR

### The Course of Study

**The leading mathematical features.** The work in this year is given to the business applications and advanced mensuration.

**Going into business.** The boy or girl should now be led to be interested in the practical business of the country, with particular reference to the common occupations of our people.

**Bank accounts.** Thrift should be encouraged and the various forms of common banking explained.

**Partial payments.** This work should be made to appear real, and the common forms of partial payment should be given instead of very difficult and unusual ones.

**Trade discount.** This should be reviewed from the standpoint of the boy or girl who is going into business and really wishes to take advantage of the discounts allowed in trade.

**Simple accounts.** Pupils should be instructed in the simple accounts of daily life, in the home, on the farm, or in the shop.

**Partnership.** The obsolete forms of this subject should give place to the real ones of to-day.

**Exchange.** The pupils, imagining themselves in business, should feel the importance of this subject from the standpoint of the merchant or of any person who orders goods

from a distance. Consider the common forms, including money orders, checks, and drafts. Simple foreign exchange. Foreign money.

**Metric system.** This should not interfere with a thorough knowledge of the common tables, and hence it is given in the last year. Its importance in our expanding trade should be recognized by teachers.

**Taxes.** Customs, state taxes, local taxes. Expenses of the government. Tax table. Assessment and collection of taxes.

**Insurance.** Fire, marine, life. Standard forms of policies.

**Corporations.** Stocks and bonds.

**Powers and roots.** Square root and its applications. Schools desiring to introduce cube root should do so at this time.

**Mensuration.** Circle, sphere, prism, cylinder, pyramid, cone.

**Nature of the problems.** The problems in this grade should relate almost exclusively to the common business of the American people of to-day. Boys and girls should thus come to know about the life they are soon to enter. Obsolete business customs should give place to information concerning the industries and occupations of our country.

**Text-book.** Smith's *Advanced Arithmetic*, Chapter II, or *Practical Arithmetic*, Chapter IV, exactly covers this work. Each furnishes a large amount of oral work, and the numerous problems for written work set forth the business conditions of the country as they are to-day. For schools wishing a sufficient amount of work, but not allowing enough time to accomplish all that is here laid down, Smith's *Grammar School Arithmetic*, Chapter III, second half, furnishes enough material. The page references in this chapter are explained on page 81 of this handbook.

## Details of the First Half of the Eighth Year's Work

**Essential features of this work.** Following this year, the boy and girl will very likely be going into such business as awaits them. It is therefore necessary to make the arithmetic as practical as possible from the standpoint of the American business man. In the first half year the most common business applications are considered. In the second half year stocks and bonds are taken up, many industrial problems are solved, powers and roots are considered, and mensuration is completed.

**Going into business** (*Adv.*, Page 138; *Prac.*, Page 400; *G. S.*, Page 306). There should be an effort at the opening of the year to make it apparent to the pupils that they may in the following year become wage-earners. The earlier pages are therefore devoted to such a class of problems as will be helpful to children who may receive the average income of beginners. This allows for a review of elementary processes, and at the same time permits the teacher to encourage the pupil in the study of elementary economics. In the *Grammar School Arithmetic* the subject of mensuration is inserted in this brief treatment of business arithmetic; but in the *Advanced Arithmetic* and the *Practical Arithmetic* this subject is postponed to the second half of the year. It is quite immaterial where the work is taken, for it has a bearing on practical life and therefore may be inserted at any time during the year.

**Bank accounts** (*Adv.*, Page 141; *Prac.*, Page 403; *G. S.*, Page 326). Even the children who have never known anything of a bank account, and these are largely in the majority, should be taught to appreciate the fact that they may soon have money enough for depositing in a savings

bank. This kind of bank is therefore first considered. Since these banks vary in different parts of the country, teachers should follow the local customs in explaining the work. The subject of compound interest will mean nothing to the child until he sees some use for it, and the first use that he will meet, and probably the only one, is in connection with the savings banks. This subject is therefore introduced in this connection. Some arithmetics give very absurd cases under this topic, such as finding the rate and the time from given conditions. Such work is so rarely met in commercial life that it should be omitted, and attention should be given to finding the compound interest itself. It is a mistake to allow pupils to feel that this subject presents any new difficulty or any particularly new mathematical feature.

Compound interest is usually computed by the help of tables. The following is part of a page from such tables, and the computation of one or two of these columns is a good exercise.

AMOUNT OF $1000 AT COMPOUND INTEREST

| YEARS | 2% | 2½% | 3% | 4% | 5% | 6% |
|---|---|---|---|---|---|---|
| 1 | 1020.00 | 1025.00 | 1030.00 | 1040.00 | 1050.00 | 1060.00 |
| 2 | 1040.40 | 1050.63 | 1060.90 | 1081.60 | 1102.50 | 1123.60 |
| 3 | 1061.21 | 1076.89 | 1092.73 | 1124.86 | 1157.63 | 1191.02 |
| 4 | 1082.43 | 1103.81 | 1125.51 | 1169.86 | 1215.51 | 1262.48 |
| 5 | 1104.08 | 1131.41 | 1159.27 | 1216.65 | 1276.28 | 1338.23 |
| 6 | 1126.16 | 1159.69 | 1194.05 | 1265.32 | 1340.10 | 1418.52 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

If the interest is at the rate of 4%, 5%, or 6% per year, but compounded semiannually, the amount is evidently the same as if the

rate were 2%, $2\frac{1}{2}$%, or 3%, respectively, compounded annually for a period twice as long.

E.g., what is the amount of $2750 for 3 yr. at 5%, compounded semiannually?

Amt. of $1000 for 6 yr. at $2\frac{1}{2}$% compounded annually = $1159.69.

" $2750 = 2.75 × $1159.69 = $3189.14.

The children should feel that they may easily acquire enough money for a small account in a savings bank. But they should also feel that they will need, if they go into business for themselves, an account in a bank of deposit. These banks are found more frequently than the savings banks, and the children should know how to make a deposit and how to draw a check. Very little mathematics is involved in this topic. It is purely a matter of business customs, and should be made as real as possible to the class. It is a good plan to establish a class bank and arrange for filling out deposit slips, depositing money, making out checks, drawing the same, and having a bank book. A limited number of the necessary blanks can be obtained in any bank, and from these the children can copy as necessary.

If a boy or girl goes into business, it may be necessary to have further relations with the banks in the way of borrowing money. The class should therefore include in their banking business the making of bank notes. Days of grace, in those states where they are still allowed, should always be included in the solution of the book or class problems.

**Annual interest.** The subject of annual interest is omitted from the text-book because in most states it is practically unknown, and it is not the business of the schools to attempt to perpetuate obsolete customs. The great advance in banking in late years has changed the methods of money

lending, and the result is that subjects like annual interest, compound interest, and partial payments with annual interest are now of relatively little value. While they may be taught for their logic, they are objectionable in many parts of the country because they give incorrect ideas of business customs. In some localities, however, annual interest is used enough to justify teachers in presenting the subject in the schools, and hence a brief treatment of it is here given.

In some states, if a note or bond contains the words "with interest payable annually," this interest, if left unpaid, also draws interest to the day of settlement, or until canceled by payment. The note or bond is then said to draw *annual interest.*

E.g., to find the amount due on a $500 note dated Jan. 1, 1906, drawing annual interest at 6%, no payments being made until the day of settlement, Jan. 1, 1910.

| | |
|---|---|
| 1. Face of note | = $500. |
| 2. Int. on $500 for 4 yr. at 6% | = 120. |
| 3. Int. on $30 for 3 yr. + 2 yr. + 1 yr. at 6% | = 10.80 |
| 4. Amt. due Jan. 1, 1904 | = $630.80 |

In step 3, one $30 payment draws interest for 3 yr., another for 2 yr., another for 1 yr., and the last one draws no interest. Therefore, there is due the equivalent of the interest on $30 for 3 yr. + 2 yr. + 1 yr.

Semiannual or quarterly interest is treated in a similar manner.

In the western states *coupon notes* are often given, that is, notes bearing coupons which are themselves promissory notes for the interest due, and also drawing interest, often at a different rate. The amount of a coupon note for $2000 at the end of 5 yr., the principal drawing 6%, the coupons representing the interest due annually and drawing 8% remaining unpaid, is easily found to be $2696.

The following exercises may be given:

*Find the amounts due on the following notes drawing annual interest, no payments being made until the day of settlement.*

1. \$1000, 4 yr., 6%, annual interest @ 6%. (*Ans.* \$1261.60.)
2. \$1200, 3 yr., 5%, annual interest @ 6%. (*Ans.* \$1390.80.)
3. \$1500, 4 yr., 6%, annual interest @ 5%. (*Ans.* \$1887.)
4. \$2000, 5 yr., 8%, annual interest @ 6%. (*Ans.* \$2896.)
5. \$2500, 5 yr., 7%, annual interest @ 6%. (*Ans.* \$3480.)
6. \$3000, 4 yr., 6%, annual interest @ 8%. (*Ans.* \$3806.40.)
7. \$1200, 4 yr., 6%, annual interest @ 6%. (*Ans.* \$1513.92.)
8. \$1414, 7 yr., 5½%, annual interest @ 5½%. (*Ans.* \$2048.22.)
9. \$1750, 4 yr., 6%, annual interest @ 6%. (*Ans.* \$2207.80.)

Several states do not allow interest above 6%.

**Discounting notes** (*Adv.*, PAGE 152; *Prac.*, PAGE 414; *G. S.*, PAGE 332). Some localities include both the first and last days in the discount period. Teachers should inquire as to the local customs and should govern themselves accordingly.

**Present worth and true discount.** Formerly this subject was included in arithmetics, but now it is generally discarded because it gives a wrong idea of business methods. The only discounts practically met to-day are bank discount (the interest paid in advance) and trade or commercial discount. For the benefit of those who may still be required to teach this subject, a brief explanation is here given.

The *present worth* of a sum due at a future date, without interest, is the sum which, at simple interest, will amount to it at the time specified.

*True discount* is the difference between a debt which does not bear interest and its present worth. It is

conveniently defined as the difference between the present and the future worth of a sum.

For example, the present worth of $212 due a year hence, allowing 6%, is $200, and $12 is the true discount.

**Partial payments** (*Adv.*, PAGE 154; *Prac.*, PAGE 416, *G. S.*, PAGE 333). In treating this subject, the attention ought to be devoted chiefly to the United States Rule. The problems here given are very practical ones, and if others are dictated by the teacher it is suggested that they should be of this nature. In general it is better to give a large number of short problems than to give a small number of very long ones. People nowadays do not make as many partial payments on promissory notes as they used to. Money is not so scarce as formerly, and if a person has an indebtedness, he is more likely to pay part and give a new note occasionally than to let the old note run a long time. The Merchants' Rule is occasionally used where the time is less than one year. It is not, however, a legal rule in most parts of the country, and therefore, if given at all, should have but little attention. It is somewhat easier than the United States Rule, but for long periods it is not so fair (*Adv.*, PAGE 157; *Prac.*, PAGE 419).

**Special rules of partial payments.** The so-called United States Rule is the one generally in use in most of the states. In some states, however, the courts have laid down other rules. These being purely local have no place in an arithmetic intended for general use. A few are inserted in this handbook for the assistance of teachers who may have occasion to use them.

In Vermont, when notes draw annual interest, and this annual interest is not fully paid when due, the procedure is as follows:

*Find the interest on each payment to the end of the yearly interest term in which it is made. Take the amount of any such payment or payments in any interest term and apply it in the following order: (1) to cancel the interest due on unpaid yearly interest; (2) to cancel unpaid yearly interest; (3) to cancel the principal.*

In New Hampshire the rule is the same as in Vermont, with the following exception:

*Payments which do not equal or exceed the interest due at the end of the year, but are intended to apply on interest not yet due, do not draw interest, but must be applied to the payment of the interest due at the end of the year.*

In Connecticut the rule is as follows:

*1. Use the United States Rule in cases where at least a year's interest has accrued when a payment is made, or when any payment is less than the interest due, or in the case of the last payment.*

*2. If less than a year's interest has accrued when the payment is made (except the last payment), take the amount of this payment to the end of the interest year and subtract this from the amount of the principal for the full interest year. Treat the remainder as the new principal.*

Where annual interest is allowed, the following plan is generally used:

*1. Find the amount of the principal (with its interest) to the end of the first interest year in which there is a partial payment. To this add the amount of any unpaid annual interest (with its interest) to the same date.*

*2. Find the amount of each payment to the same date.*

*3. Take the difference between these two sums as a new principal and proceed as before.*

In case this new principal exceeds the preceding one, that is, if the amount of the payments does not cancel all accrued interest, the unpaid interest draws only simple (not compound) interest to the end of the next interest year in which a payment is made.

For example, a note for $400 is dated May 1, 1907, and is due in 3 yr. at 6% annual interest. It has the following partial payments indorsed: Nov. 1, 1908, $150; Nov. 1, 1909, $100. Required the amount due May 1, 1910.

| | | |
|---|---|---|
| First principal | $400.00 | |
| Int. to May 1, 1909 | 48.00 | |
| Int. on first annual int. | 1.48 | |
| Amount May 1, 1909 | | $449.44 |
| First payment | $150.00 | |
| Int. on it, ½ yr. | 4.50 | |
| | | $154.50 |
| New principal | | $294.94 |
| New principal | $294.94 | |
| Int. to May 1, 1910 | 17.70 | |
| | | $312.64 |
| Second payment | $100.00 | |
| Int. on it, ½ yr. | 3.00 | |
| | | $103.00 |
| Amt. due May 1, 1910 | | $209.64 |

**Trade discount** (*Adv.*, PAGE 158; *Prac.*, PAGE 420; *G. S.*, PAGE 335). This is one of the most important applications of percentage. In bargain sales where discounts of 10%, 20%, or 30% are allowed, very many people come in contact with it. Of course every merchant in buying goods has occasion to know about it. Therefore considerable attention is paid to this subject, the problems being very practical ones.

Since the children will need to order goods at some time, even though they are not in trade, it is well that they should be drilled upon model orders and model bills (*Adv.*, PAGE 160; *Prac.*, PAGE 422; *G. S.*, PAGE 337). There is a good deal of complaint that children coming from the grammar school cannot write a good business letter. It is therefore suggested that much practice be given in this work in connection with the lessons in writing.

**Simple accounts** (*Adv.*, PAGE 162; *Prac.*, PAGE 424; *G. S.*, PAGE 338). Very few pupils need to know about double-entry bookkeeping. Every one, however, needs to know about keeping a simple account. This work involves practically no elaborate features in arithmetic, being quite as much a matter of penmanship. Sufficient work is given in the author's books to show the forms that are recommended. It is a valuable exercise for the pupils to make imaginary accounts in connection with their work in penmanship.

**Partnership** (*Adv.*, PAGE 164; *Prac.*, PAGE 426; *G. S.*, PAGE 339). This subject formerly occupied a great deal of time in arithmetic, and involved a case which has long been substantially obsolete. This case is where the partners put in different sums of money for different lengths of time, the problems then becoming quite complex. But little of this work is given in the author's books, the examples being as nearly genuine as possible. They include every important principle, but neglect such obsolete ones as would give a wrong notion of business.

**Exchange** (*Adv.*, PAGE 167; *Prac.*, PAGE 429; *G. S.*, PAGE 341). This subject has changed greatly during the past few years. The postal and express money orders have materially modified the paying of bills at a distance. The author's books make this subject very practical by showing the actual forms of exchange in use at the present

time. The bank draft is usually employed for large sums of money, and the pupil should be given a good deal of exercise in properly filling out these drafts. Most arithmetics do not distinguish between the bank draft and the commercial draft. This is clearly done in the author's books.

The clearing house (*Adv.*, PAGE 172; *Prac.*, PAGE 434; *G. S.*, PAGE 345) is to be found in all large cities. Here representatives of the banks meet every morning, and each bank presents the checks and drafts that it holds to the bank upon which they were drawn. The clerks make out a list of the drafts and checks that are deposited, and also a list of those that are received. Of course the total amount drawn upon all the banks together must equal the total amount received by them. Each bank then finds the amount that it owes and the amount that is due it. If it owes more than is due, it pays that money into the clearing house. If it has more due it than it owes, the clearing house pays that amount. Of course the result is that the clearing house just balances after all these transactions have been made. Such a plan saves the trouble and risk of sending a large amount of money about the city, and also makes it possible for banks to settle large accounts with only a small amount of actual cash. For example, if a bank has $200,000 due it, and owes the other banks of the clearing house $190,000, the whole thing is settled by an actual transfer of only $10,000, instead of $390,000.

The subject of domestic (United States) exchange is not of enough importance to make it worth while to dwell longer upon it than is necessary for covering the work laid down in the author's books.

The subject of foreign exchange is less important, for the reason that very few of the pupils will ever have

occasion to use it. It is part of our national business, however, to buy and sell goods abroad, and this business is growing rapidly. On this account some attention to the subject is necessary. The table of foreign money has very little meaning to pupils unless taken up in this connection. To put this work back in compound numbers is to make it seem very unreal.

**Metric system** (*Adv.*, PAGE 178; *Prac.*, PAGE 440; *G. S.*, PAGE 349). There is sometimes a disposition to teach this subject very early in the grades. One reason for so doing is that it is an easy subject, involving nothing but decimal fractions. Another reason is that pupils will remember it more easily if it is learned at that time. One answer to this argument is that the value of the system will not be appreciated until the pupils begin to see some application for it. This application will be found in the work in science in the eighth grade or the high school, or in the study of our foreign commerce which belongs to the eighth school year. It will be found helpful to interest the pupils in the history of this valuable system. It arose in France about the year 1800. The people, however, were so attached to their old system, which was quite as difficult as the one we use in America, that it was not until about the year 1840 that it was made compulsory in France. Since that time, and more particularly since 1880, it has been very generally adopted in much of the civilized world, excepting in English-speaking countries. In the United States it has grown rapidly since that time, and is now used in all scientific work in our laboratories. Many physicians, especially in the cities, use it instead of the old cumbersome table of apothecaries' measures. The recent advance of the United States in the matter of foreign trade is making it necessary to use these measures in the building

of machinery that is to be sold abroad. The result will be that the system will be needed by those who are going to enter this field. While it will not take the place of our present system for a long time to come, it is going to grow constantly, and even now it is necessary that every fairly educated person should understand it. The system, however, will not be appreciated by children, or by teachers, unless the actual measures are in the hands of the class, and are really used. These measures are easily made, and there is no excuse for failing to make the subject seem genuine to those who study it. The work should be largely oral, of the kind suggested in the author's books. There is not much need for elaborate written work in such a subject.

A few examples in the chapter on the metric system involve specific gravity (*Adv.*, PAGE 184; *Prac.*, PAGE 446). For further information concerning this subject, see page 75 of this handbook.

**Taxes** (*Adv.*, PAGE 190; *Prac.*, PAGE 452; *G. S.*, PAGE 354). In order to make the subject of taxes real, the pupils must know something about the expenses of our government. For this reason a considerable amount of work of this kind is inserted, particularly in the *Advanced Arithmetic* and the *Practical Arithmetic*. This offers a good opportunity to explain to the children that the United States cannot levy direct taxes, but that its income is derived from duties on imports, from the sale of revenue stamps and postage stamps, and from other similar sources. In the matter of state and local taxes, the first point to be considered is the meaning of a tax and the method of levying it. A very good exercise in computing consists in preparing a small tax table with a rate different from the one given in the book.

Some arithmetics give a rule for finding the collector's commission which is different from that given in the author's books. The latter, however (*Adv.*, PAGE 200; *Prac.*, PAGE 462; *G. S.*, PAGE 358), is the correct one and is used by all collectors. In the examples given, the collector's commission is really 45.38¢, but the collectors take 46¢. Indeed, the rule frequently laid down, that a sum less than ½¢ is rejected, does not usually apply in the making out of bills or in computing commission. Whatever fraction of a cent there may be in such cases is usually counted a cent.

Poll tax should be emphasized in those sections of the country in which it is an important feature, but not elsewhere. In the cities it is often no longer considered.

**Fire insurance** (*Adv.*, PAGE 201; *Prac.*, PAGE 463; *G. S.*, PAGE 359). This subject offers no new principles from the mathematical standpoint. The meaning of rate, as a certain sum for each hundred dollars, is the only essentially new feature. As to the length of time for which property is insured, teachers should be governed by local conditions. In some portions of the country houses are insured for five years, but this is not the general custom.

**Life insurance** (*Adv.*, PAGE 206; *Prac.*, PAGE 468; *G. S.*, PAGE 361). This subject is so extensive and has become so complex that it is impossible to give it a very complete treatment in arithmetic. Neither is it necessary to do so, for the reason that a citizen ordinarily needs to know only the leading forms of policies and the meaning of rate as so much on $1000. This being understood, the subject presents no difficulties, unless one goes into the mathematical details as an actuary, and this of course cannot be done in the schools.

## Details of the Second Half of the Eighth Year's Work

**Corporations** (*Adv.*, Page 210; *Prac.*, Page 472; *G. S.*, Page 362). Every citizen at the present time needs to know about the forming of corporations. Even though he may not have any stock in a company, he will come into contact with these great organizations. The whole matter of trusts and monopolies is involved in this subject. Teachers can easily amplify the work given in any arithmetic, and with a good class it will be advantageous to do so. It is frequently advisable to have the class form itself into a corporation, electing its officers and directors and issuing stock and bonds. It is exceedingly desirable that children should not get the wrong idea that dealing in stocks and bonds is necessarily gambling, or that it need be in any way illegitimate. Many poor people invest their money in stocks, and this investment is entirely legitimate. The subject seems more real to the class if the stock quotations are taken from a daily newspaper. Since it is invariably the case that the broker's commission must be considered, this feature should not in general be omitted.

**Buying produce** (*Adv.*, Page 219; *Prac.*, Page 481; *G. S.*, Page 367). A little of this work is desirable in any locality. In a rural community it shows the way in which the general products are sold in large quantities. In the cities it shows the source of much of the food supply. It is not desirable, however, that children should be compelled to learn the technicalities of trade as here presented, but should be allowed to refer to the text-book for such as are necessary for the solutions.

**Industrial problems** (*Adv.*, Page 221; *Prac.*, Page 483; *G. S.*, Page 369). These problems refer to some of the

large industries of our country. They are of the most practical nature and relate to the interests of a large number of people. It is wise, however, to supplement this work by a reference to the local industries.

**Powers and roots** (*Adv.*, PAGE 230; *Prac.*, PAGE 492; *G. S.*, PAGE 311). This subject may be taken earlier in the year if desired. In the *Grammar School Arithmetic* it is given somewhat early, thus correlating more with the algebra work, if the author's *Algebra for Beginners* is taken during this year. The subject of square root has not very many applications in practical work. Those who have occasion to extract roots very often, as in bridge building or other forms of engineering, commonly use tables in which the roots of the numbers are given. The one application in which children have any interest is that of finding the length of the hypotenuse. A considerable amount of such work is given. The explanation of square root as given in the author's arithmetics is complete enough to render reproduction in this manual unnecessary. The subject is taught chiefly as an excellent example in logic, and it is therefore not advisable to treat it merely as a matter of rule.

Cube root is taken up in the *Advanced Arithmetic* and the *Practical Arithmetic*. It is not given in the *Grammar School Arithmetic* for the reason that the subject is being omitted quite generally from courses. Teachers are advised to postpone this subject until algebra is taken up, unless the time assigned to arithmetic is such as to warrant its treatment here. It has very little practical value, and as an example in logic it belongs quite as properly in the high school as in the grades.

**Mensuration** (*Adv.*, PAGE 248; *Prac.*, PAGE 512; *G. S.*, PAGE 308). This subject may also be taken earlier in the year if desired. It is given earlier in the *Grammar*

*School Arithmetic*, so as to bring it closely in connection with square root, and thus to correlate it with the algebra work. It is recommended that the models be used as suggested in the book. These can easily be made by the pupils out of paper, and this is one of the best ways of procuring them. Avoid mere arbitrary rules, for unless the objects are in the hands of the children the subject will not be understood by them and the necessary rules will soon be forgotten.

It is not advisable to introduce work with other figures than those given in these books. For example, the frustum of a pyramid and that of a cone are not needed by the ordinary citizen, and work with these solids may well be left to the classes in geometry. Since, however, there is an occasional demand for some of these cases, a few rules are here given, to be explained by the teacher and used as necessary.

*Regular polygon.* The area equals the perimeter multiplied by half the apothem (distance from the center to one of the sides).

*Circle.* The diameter equals the circumference multiplied by 0.3183+.

The area equals the square of the diameter multiplied by 0.7854−.

The side of an inscribed square equals the diameter multiplied by 0.7071+, or the circumference multiplied by 0.2251−.

*Ellipse.* The area equals the product of the semiaxes, multiplied by $\pi$.

*Frustum of a pyramid or cone.* The convex surface of a frustum of a regular pyramid or cone equals the sum of the perimeters of the bases multiplied by half the slant height.

The volume of frustum of height $h$ and bases $b$ and $b'$ equals $\frac{h}{3}(b + b' + \sqrt{bb'})$.

*Sphere.* The volume may be found by multiplying the cube of the diameter by 0.5236–.

The edge of a cube inscribed in a sphere of diameter $d$ is $d \div \sqrt{3}$.

*Doyle's rule for lumber measure.* From the diameter of a log, in inches, subtract 4, and square the remainder. The result will be the number of square feet of inch boards in a 16-ft. log.

**Average of payments and accounts.** It was formerly the custom in certain lines of business to have long-standing accounts between a debtor and creditor. When these accounts were settled there arose the question of an equitable adjustment of interest, giving rise to a subject often known as *Equation of Payments.* Although this custom is still observed in Europe in certain cases, it has become obsolete in America. The teaching of the subject therefore does more harm by giving a wrong idea of business than good by the mental discipline involved. Accounts are now generally settled within a limited time, discounts being allowed for promptness of payment.

A brief statement of the matter is here inserted for the benefit of teachers who may be asked about it, or who may feel that something is gained by teaching it.

Finding the time at which several payments, due at different times, may be settled without injustice to either party, is called the *averaging of payments*, or finding the *average term of credit*, or finding the *equated time of payment.*

E.g. A owes B $20 due to-day, $15 due in 2 mo., and $20 due in 4 mo. At what time may the whole indebtedness be discharged by one payment?

**The interest on \$15, for example, for 2 mo., is the same as that on \$1 for 15 × 2 mo., or 30 mo. Applying this principle, we have the following:**

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **The** | **use** | **of** | **\$20** | **for** | **0** | **mo.** | **=** | **the** | **use** | **of** | **\$1** | **for** | **0** | **mo.** |
| " | " | " | **\$15** | " | **2** | " | **=** | " | " | " | " | " | **30** | " |
| " | " | " | **\$20** | " | **4** | " | **=** | " | " | " | " | " | **80** | " |
| " | " | " | **\$55** | " | $x$ | " | **=** | " | " | " | " | " | **110** | " |

**Therefore** $55x = 110$, $x = 2$.
**That is, the entire debt may equitably be discharged in 2 mo.**

From this example we see that the following is true:

*To find the average term of credit, multiply each debt by its term of credit and divide the sum of the products by the sum of the debts.*

The following examples may be used if desired:

1. A owes B \$30 due in 4 mo., \$40 due in 5 mo., and \$80 due in 6 mo. What is the average term of credit?
*Ans.* 5 mo. 10 da.

2. C owes D \$400 due in 4 mo., \$600 due in 6 mo., and \$200 due in 9 mo. What is the average term of credit?
*Ans.* 5 mo. 25 da.

3. E owes F \$100 due in 2 mo., \$150 due in 4 mo., \$90 due in 6 mo., and \$300 due in 12 mo. What is the average term of credit? *Ans.* 7 mo. 22 da.

4. G owes H \$3000 due in 2 mo., \$5000 due in 4 mo., and \$2000 due in 6 mo. What is the average term of credit? *Ans.* 3 mo. 24 da.

5. X owes Y \$850 due to-day, \$300 due in 6 mo., and \$550 due in 9 mo. What is the equated time?
*Ans.* 3 mo. 29 da. after to-day.

6. On January 15 I bought of Gorham & Co. a bill of goods amounting to $50, on 2 mo. credit; on February 1, $35, on 3 mo. credit; on February 19, $40, on 2 mo. credit; and on March 17, $38, on 4 mo. credit. What is the equated time? *Ans.* 48 da. after March 15, or May 2.

When an account contains both debits and credits we proceed as in the following example:

| *Bought* | | | *Paid* | | |
|---|---|---|---|---|---|
| June 7, | Merchandise, | $106. | July 22, | Cash, | $96 |
| Sept. 7, | " | 85.60 | Aug. 15, | " | 46 |
| Oct. 23, | " | 88. | Oct. 3, | " | 28 |

We may find when the balance should be paid by computing from the first date, or from the last one. Indeed, we might take any date as one from which to work. Suppose we take the last one, October 23. The buyer will then have had the use of the goods worth $106 for 138 da. (See table of time, *Adv.*, page 105, for such work.) On the other hand, the seller will have had the use of $96 from July 22 to October 23, 93 da., equivalent to the use of $1 for 8928 da. Therefore we have the following:

| *Bought* Items | Time | $1 for | *Paid* Items | Time | $1 for |
|---|---|---|---|---|---|
| $106. | 138 da. | 14628 da. | $96 | 93 | 8928 da. |
| 85.60 | 46 " | 3938 " | 46 | 69 | 3174 " |
| 88. | 0 " | 0 " | 28 | 20 | 560 " |
| $279.60 | | 18566 " | $170 | | 12662 " |
| 170. | | 12662 " | | | |
| $109.60 | | 5904 " | | | |

$$5904 \text{ da.} \div 109.60 = 54 \text{ da.}$$

Therefore a note for the balance, $109.60, should be dated 54 da. before October 23, or August 31.

1. I owe $100 April 10. If I pay $56 March 16, and $32.40 April 6, when should I pay the rest?

*Ans.* August 15.

2. I owe $75 due March 1. On January 1 of the same year I paid $50. When should the balance be paid?

*Ans.* July 1.

3. I owe $50 due March 1. On January 1 of the same year I paid $75, and thus have $25 coming to me. When should a note for this balance be dated?

*Ans.* September 1, last year.

4. I owe $30 due March 7, $47.40 due May 10, $56.40 due June 17. I have paid $24.70 March 18, $40 May 12, and $26 July 8. When should a note for the balance be dated? *Ans.* May 8.

## A Course in the Beginning of Algebra

*(Adapted to the seventh or eighth grade, or to the first year in the high school)*

**The leading mathematical features.** These are the fundamental operations, the linear or simple equation with integers and fractions, simultaneous equations, and an introduction to quadratics. The work should be arranged with reference to a proper psychological sequence.

**Some of the uses of algebra.** The pupil should be at once shown the value of the subject in the understanding of formulas, in solving the linear equation, and in the simplification of the solution of problems already met in arithmetic.

**Terms explained.** Omit all terms that are not actually used at once. Show the uses of monomials and polynomials.

**The negative number.** Introduce it after the pupil has learned to appreciate the value of algebra. Introduce curve tracing as a means of understanding statistics.

**The fundamental operations.** In the first part of the course, these should be introduced in an elementary way, the multipliers and divisors being monomials only, and factoring being limited to polynomials having two factors, one being a monomial. In the latter part of the course, the work should be extended, thus treating algebra as modern arithmetic is treated, arranging it on the basis of difficulty rather than on that of the operations.

**Factoring.** Limit the cases to those actually needed in reducing fractions to lower terms, or in solving the quadratic.

**Fractions.** In the first part of the year take the four operations with fractions having monomial denominators, extending the work in the second half year.

**Linear or simple equations.** Easy linear equations with one unknown quantity should appear in the first half of the course. Simultaneous linear equations with two unknowns should be reserved for the second half of the course.

**Quadratic equations.** Easy cases solved by factoring should be introduced in the second half of the course.

**Roots.** The work in such an elementary course should be limited to the square roots of numbers and of square polynomials.

**Problems.** These should be made as practical as possible, showing the value of algebra in our daily life. At the same time it must be remembered that most of the work in algebra is necessarily with abstract forms.

**Oral problems.** Oral problems should be emphasized, as in arithmetic.

**Text-book.** Smith's *Algebra for Beginners* exactly covers this work.